WORKSHOP ON TRANSPORT AND LOCALIZATION IN RANDOM MEDIA: THEORY AND APPLICATIONS, MAY 1ST - MAY 3RD 2018

Registration and name tag distribution start on Tuesday at 9am in the Mathematics Lounge, room 508 Mathematics.

The talks all take place in the Mathematics Department, room 312 Mathematics.

Breakfast and Tea take place in room 508 Mathematics.

There will be a reception with a dinner in room 508 Mathematics at 6pm on Tuesday.

	Tuesday	Wednesday	Thursday
9 – 10	Breakfast	Breakfast	Breakfast
10 – 11	Spencer	Bal	Serfaty
11 : 30 – 12 : 30	Amstrong	Warzel	Mattingly
12 : 30 – 2	Lunch	Lunch	Lunch
2 – 3	Garnier	Bramson	Lu
3 : 15 – 4 : 15	Ryzhik	Hislop	Borcea
4 : 15 – 5	Tea	Tea	Tea
5 – 6	Jitomirskaya	Nolen	Papanicolaou

Abstracts

Scott Amstrong: Homogenization and regularity for nonlinear elliptic equations.

I will give an overview of some recent quantitative results for linear elliptic PDEs with random coefficients, with particular focus on the role played by the "large-scale" regularity theory. I will then describe the challenges in extending the methods to nonlinear equations and some recent partial results in this direction.

Guillaume Bal: Topological protection of perturbed edge states.

Topological insulators are materials characterized by a non-trivial topological invariant. A striking feature is the presence of edge states at the interface between two topologically distinct materials that propagate in a privileged direction. Since this asymmetric transport is protected topologically, it is immune to random fluctuations that do not break the invariant. The resulting absence of back-scattering makes these materials appealing practically.

We model such two-dimensional materials by systems of Dirac equations. Topological invariants in the form of Fredholm indices can be assigned to these systems to describe the asymmetric edge states. We develop a scattering theory to assess the quantitative influence of random fluctuations on transport along the edge. In the diffusive regime, we obtain a configuration where transport and (Anderson) localization coexist: a number of modes described by topology transport while all other modes localize. The results also generalize to the setting of fermionic time reversal symmetry, where the standard index is replaced by a $\mathbb{Z}_2$ index.

Liliana Borcea: Wave propagation in random waveguides with turning points.

Guided waves arise in a variety of applications like underwater acoustics, optics, the design of musical instruments, and so on. We present an analysis of wave propagation and reflection in an acoustic waveguide with random sound soft boundary and a turning point. The waveguide has slowly bending axis and variable cross section. The variation consists of a slow and monotone change of the width of the waveguide and small and rapid fluctuations of the boundary, on the scale of the wavelength. These fluctuations are modeled as random. The turning point is many wavelengths away from the source, which emits a pulse that propagates toward the turning point, where it is reflected. We consider a regime where scattering at the random boundary has a significant effect on the reflected pulse. We determine from first principles when this effects amounts to a deterministic pulse deformation. This is known as a pulse stabilization result. The reflected pulse shape is not the same as the emitted one. It is damped, due to scattering at the boundary, and is deformed by dispersion in the waveguide. An example of an application of this result is in inverse problems, where the travel time of reflected pulses at the turning points can be used to determine the geometry of the waveguide.

Maury Bramson: Majority vote processes on trees

The majority vote process was one of the first interacting particle systems to be investigated and can be described as follows. There are two possible opinions at each site and that opinion switches randomly to the majority opinion of the neighboring sites. Also, at a different rate ε, the opinion at each site randomly changes due to noise.

Despite its simple dynamics, the majority vote process is difficult to analyze. In particular, on $\mathbb{Z}^d$ with $d > 1$ and ε chosen small, it is not known whether there exists more than one equilibrium. This is surprising due to the close analogy between the majority vote process and the Ising model.

Here, we discuss work with Larry Gray on the majority vote process on the infinite tree with vertex degree d, where it is shown that, for small noise, there are uncountably many mutually singular equilibria, and that convergence to equilibrium occurs exponentially quickly from nearby initial states. Our methods rely on graphical constructions; they are quite flexible and can be used to obtain analogous results for other models, such as the stochastic Ising model on a tree.

Josselin Garnier: Imaging through random media by speckle intensity correlations. When waves propagate through random media the energy is transferred to the incoherent wave part by scattering. The wave intensity then forms a random speckle pattern seemingly without much useful information. However, a number of recent physical experiments show that it is possible to extract useful information from this speckle pattern and to image an object buried in a random medium. Here we present the mathematical analysis that explains the quite stunning performance of speckle imaging. Our analysis identifies a scaling regime where these schemes work well. This regime is the white-noise paraxial regime, which leads to the Ito-Schrodinger model for the wave amplitude. Our results conform with the sophisticated physical intuition that has motivated these schemes, but give a more detailed characterization of the performance. The analysis gives a description of (i) the information that can be extracted and with what resolution (ii) the statistical stability or signal-to-noise ratio with which the information can be extracted. This is a joint work with Knut Solna (UC Irvine).

Jianfeng Lu: Optimal artificial boundary condition for random elliptic media. We are given a uniformly elliptic coefficient field that we regard as a realization of a stationary and finite-range (say, range unity) ensemble of coefficient fields. Given a (deterministic) right-hand-side supported in a ball of size $\ell \gg 1$ and of vanishing average, we are interested in an algorithm to compute the (gradient of the) solution near the origin, just using the knowledge of the (given realization of the) coefficient field in some large box of size $L \gg \ell$. More precisely, we are interested in the most seamless (artificial) boundary condition on the boundary of the computational domain of size L.

Our algorithm is motivated by the recently introduced multipole expansion in random media. We rigorously establish an error estimate (on the level of the gradient) in terms of $L \gg \ell \gg 1$, using recent results in quantitative stochastic homogenization. More precisely, our error estimate has an a priori and an a posteriori aspect: With a priori overwhelming probability, the (random) prefactor can be bounded by a constant that is computable without much further effort, on the basis of the given realization in the box of size L.

We also rigorously establish that the order of the error estimate in both L and ℓ is optimal. This amounts to a lower bound on the variance of the quantity of interest when conditioned on the coefficients inside the computational domain, and relies on the deterministic insight that a sensitivity analysis wrt a defect commutes with (stochastic) homogenization. Numerical experiments show that the optimal convergence rate already sets in at only moderately large L, and that more naive boundary conditions perform worse both in terms of rate and prefactor. Joint work with Felix Otto.

Peter Hislop: Localization and spectral statistics for Schrödinger operators with random point interactions. We discuss localization and local eigenvalue statistics for Schrödinger operators with random point interactions on $\mathbb{R}^d$, for $d = 1, 2, 3$. The results rely on probabilistic estimates, such as the Wegner and Minami estimate, for the eigenvalues of the Schrödinger operator restricted to cubes. The special structure of the point interactions facilitates the proofs of these eigenvalue correlation estimates. One of the main results is that the local eigenvalue statistics is given by a Poisson point process in the localization regime, one of the first examples of Poisson eigenvalue statistics for multi-dimensional random Schrödinger operators in the continuum. This is joint work with M. Krishna and W. Kirsch.

Svetlana Jitomirskaya: Arithmetic transitions in transport and localization for quasiperiodic operators.

A very captivating question in solid state physics is to determine/understand the hierarchical structure of spectral features of operators describing 2D Bloch electrons in perpendicular magnetic fields, as related to the continued fraction expansion of the magnetic flux. In particular, the hierarchical behavior of the eigenfunctions of the almost Mathieu operators, despite significant numerical studies and even a discovery of Bethe Ansatz solutions has remained an important open challenge even at the physics level.

I will present a complete solution of this problem in the exponential sense throughout the entire localization regime. Namely, I will describe, with very high precision, the continued fraction driven hierarchy of local maxima, and a universal (also continued fraction expansion dependent) function that determines local behavior of all eigenfunctions around each maximum, thus giving a complete and precise description of the hierarchical structure. In the regime of Diophantine frequencies and phase resonances there is another universal function that governs the behavior around the local maxima, and a reflective-hierarchical structure of those, phenomena not even described in the physics literature. These results lead also to the proof of sharp arithmetic transitions between pure point and singular continuous spectrum, in both frequency and phase, as conjectured since 1994.

In the singular continuous regime, the hierarchical structure of generalized eigenfunctions is responsible for unusual quantum dynamics, leading to arithmetic transitions in local dimensions and transport.

The above singular continuous results are not sharp but hold for general analytic quasiperiodic potentials. I will also describe a sharp arithmetic transition in spectral dimensions and transport holding for this general class. The talk is based on papers joint with W. Liu, S. Tcheremchantsev, and S. Zhang.

Jonathan Mattingly: TBA.

Jim Nolen: A variance lower bound for the effective conductivity in random homogenization.

Consider the effective conductivity matrix for a sample of random material having size $L > 0$, where the random conductivity is stationary and has short-range dependence. As L increases to infinity, the finite-sample effective conductivity A_L converges to a deterministic matrix A, the homogenized conductivity coefficient. Gloria and Otto proved an upper bound on the variance of A_L of the order L^d (i.e. the volume), as would be the case for a sum of L^d independent random variables. In this talk, I'll describe a sufficient condition on the coefficient field which implies that an $O(L^d)$ lower bound also holds. Examples will be given, illustrating the use of the sufficient condition for different random conductivity laws. In deriving this result, key ideas are (1) the coupling of laws of the coefficient matrix and (2) understanding the effect of a large inclusion in an otherwise homogeneous background field. This variance lower bound plays a role in normal approximation (via Stein's method) for the random variable A_L. This is joint work with Felix Otto.

Georges Papanicolaou: Imaging in Random Media.

An ambient medium that is inhomogeneous will make imaging of an object in it difficult if not impossible. I will review the role of inhomogeneity in imaging and how it can be addressed, starting with very weak inhomogeneities and correlation based imaging and advancing into more complex ambient media where considerable phase randomization occurs so that phase reconstruction is needed.

A variety of mathematical methods can be used in this context and I will review them as well, up to some recent advances that exploit illumination redundancy and holography.

Lenya Ryzhik: The random heat equation in dimensions three and higher

We consider the heat equation with a multiplicative Gaussian potential in dimensions three and higher. We show that the renormalized solution converges to the solution of a deterministic diffusion equation with an effective diffusivity. We also prove that the renormalized large scale random fluctuations are described by the Edwards–Wilkinson model, that is, the stochastic heat equation (SHE) with an additive white noise, with an effective diffusion and an effective variance. This is a joint work with Alex Dunlap, Yu Gu and Ofer Zeitouni.

Sylvia Serfaty: Mean-Field Limits for Ginzburg–Landau vortices and related problem.

Ginzburg–Landau type equations are models for superconductivity, superfluidity, Bose–Einstein condensation. A crucial feature is the presence of quantized vortices, which are topological zeroes of the complex-valued solutions, and which interact like Coulomb particles. We will present new results on the derivation of mean field limits for the dynamics of many vortices starting from the parabolic Ginzburg–Landau equation or the Gross-Pitaevskii (= Schrödinger Ginzburg–Landau) equation, as well as results and questions on the situation with random environment, and similar results for the discrete problem of Coulomb-interacting particles.

Thomas Spencer: Two Classical models of Quantum Dynamics.

The talk will start with an elementary introduction to the quantum dynamics of a Schrödinger operator with a random potential. Certain features of this model are believed to be reflected in classical models. In two dimensions one such model is described by the motion of a classical particle on the Manhattan lattice which is deflected by randomly placed obstructions. The second model is the linearly edge reinforced random walk on a d dimensional lattice. In three dimensions this model has a localization - diffusion transition as one varies the strength of reinforcement.

Simone Warzel: Localization-delocalization transitions in random matrix models: a SPDE approach.

Hermitian random matrix models are known to exhibit phase transitions regarding both their local eigenvalue statistics as well as the localisation properties of their eigenvectors. The poster child of such a model is the Rosenzweig–Porter model, i.e. the interpolation of a random diagonal matrix and GOE. Interestingly, this model has recently been shown to exhibit a phase in which the eigenvectors exhibit non-ergodic delocalisation alongside the local GOE statistics. In this talk, I will explain the main ideas behind the emergence of this phase using a SPDE approach. Time permitting, I will also address the motivation for these questions and consequences for the ultra-metric ensemble. (The talk is based on joint works with Per von Soosten.)