Analysis and Computation in Kinetic Theory
Stanford University
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Abstracts

Guillaume Bal

Inverse problems and boundary controls for kinetic transport equations

This talk will address several inverse problems in linear kinetic theory. It will revisit the reconstruction of the kinetic coefficients from boundary measurements and associated stability estimates. It will also present recent results on the control of internal transport solutions or outgoing solutions by means of judiciously chosen incoming conditions. In elliptic problems, for instance arising as highly scattering limits of kinetic models, such internal controls are primarily based on applications of the unique continuation property. We will show that the latter does not hold for kinetic equations and will present situations where such controls are feasible or not.

Vincent Calvez

Concentration waves of bacteria at the mesoscopic scale.

Solitary waves of swimming bacteria Escherichia coli were described in the seminal paper by Adler (Science 1966). These experiments gave rise to intensive PDE modelling and analysis, after the original model by Keller and Segel (J. Theor. Biol. 1971). Together with Bournaveas, Perthame, Raoul and Schmeiser, we have revisited this old problem from the point of view of kinetic transport equations. This framework is very much adapted to the so-called run-and-tumble motion, in which any bacteria modulate the frequency of reorientation (tumble) -- and thus the duration of free runs -- depending on chemical variations in its environment.

In this talk, I will present existence results for solitary waves both at the macroscopic scale, and at the mesoscopic scale. The macroscopic problem consists of a drift-diffusion equation derived from the kinetic equation after a suitable diffusive rescaling, coupled to two reaction-diffusion equations. Unexpected mathematical difficulties arise at the mesoscopic scale, where the unconditional proof of existence of travelling waves require a refined description of spatial and velocity profiles.

Irene Gamba Conservative schemes for Vlasov-Poisson/Maxwell Landau systems We discuss a hybrid numerical approach to Vlasov-Poisson-Landau for plasma systems. We propose a time splitting computational scheme that allows for a conservative approach for the whole system, where the Landau operator is computed by an spectral conservative based solver, and both, the Vlasov-Poisson and Vlasov-Maxwell systems, are computed by a conservative Discontinuous Galerkin approach.
Pierre-Emmanuel Jabin Weak Solutions for Compresssible Navier-Stokes Equations: Thermodynamically unstable pressure and anisotropic viscous stress tensor We prove global existence of appropriate weak solutions for the compressible Navier-Stokes equations for more general stress tensor than those covered by P.-L. Lions and E. Feireisl's theory. More precisely we focus on more general pressure laws which are not thermodynamically stable; we are also able to handle some anisotropy in the viscous stress tensor. To give answers to these two longstanding problems, we revisit the classical compactness theory on the density by obtaining precise quantitative regularity estimates: This requires a more precise analysis of the structure of the equations combined to a novel approach to the compactness of the continuity equation. These two cases open the theory to important physical applications, for instance to describe solar events (virial pressure law), geophysical flows (eddy viscosity) or biological situations (anisotropy). Joint work in collaboration with D. Bresch".
Shi Jin Semiclassical computational methods for quantum dynamics with band-crossing Band-crossing is a quantum dynamical behavior that contributes to important physics and chemistry phenomena such as quantum tunneling, Berry connection, chemical reaction etc. In this talk, we will discuss several recent works in developing semiclassical methods for band-crossing, including examples from surface hopping, Schrodinger equation with periodic potentials, and high frequency solutions of linear hyperbolic systems with polarized waves. For such systems we will also introduce an "asymptotic-preserving" method that is accurate uniformly for all wave numbers.

Sasha Kiselev Small scale creation in ideal fluids It has been known since 1930s that incompressible 2D Euler equation is globally regular. The best upper bound on the growth of the derivatives of vorticity is double exponential in time. I will describe a construction showing that such fast growth can actually happen, so that the double exponential bound is qualitatively sharp. This is based on a joint work with Vladimir Sverak. Our work has been motivated by numerical experiments due to Hou and Luo who propose a new scenario for singularity formation in solutions of 3D Euler equation. The scenario is axi-symmetric and so effectively two-dimensional. The geometry of the scenario is similar to the geometry of our 2D Euler example and involves hyperbolic points of the flow located on the boundary of the domain. Provided I have time, I will discuss two one-dimensional models that have been developed to gain insight into the behavior of solutions in the 3D setting.
Jianguo Liu An analysis of merging-splitting dynamics by Bernstein function theory We study coagulation-fragmentation equations inspired by data analysis in fisheries science that shows a universal size distribution for schools of fish in the mid-ocean. Although the equations lack detailed balance and admit no H-theorem, we are able to develop a rather complete description of equilibrium profiles and large-time behavior, based on complex function theory for Bernstein and Pick functions. The generating function for discrete equilibrium profiles also generates the Fuss-Catalan numbers (derived by Lambert in 1758) that count all ternary trees with n nodes. The structure of equilibrium profiles and other related sequences is explained through a new and elegant characterization of the generating functions of completely monotone sequences as those Pick functions analytic and nonnegative on $(-\infty,1)$. This is joint work with Bob Pego and Pierre Degond.
Tai-Ping Liu Boltzmann equation for Infinite range potentials. In collaboration with Jin-Cheng Jiang, we study the Boltzmann equation with infinite range potentials. We construct the collision operator when the potential is taken as the limit of cut-off, finite range potentials. We show that this process yields the classical collision operator as proposed by Maxwell.

Jianfeng Lu

Solving linear half-space kinetic equations with general boundary conditions

In this talk, we will discuss some recent progress on efficient methods for solving linear / linearized half-space kinetic equations. The spectral method we recently develop relies on the ideas of even-odd decomposition and damping adding / removal. The method works for a large class of equations with general boundary conditions.

Benoit Perthame

Kinetic equations for bacterial movements: from molecular pathways to macroscopic equations

Self-organisation of cell colonies is of paramount importance in medecine and biology (biofilms, bacterial invasions, tissue growth). For `simple' organisms as bacteria, a large variety of chemical processes and biophysical laws can be involved in communications between cells. An example is chemotaxis when cells interact through a chemoattracting signal; In their seminal work, E. Keller and L. Segel proposed the first description using a parabolic system that can exhibit various qualitative behaviors as dispersion or blow-up. Recently hyperbolic models have been proposed in order to better take into account the smallness of the cell diffusion, the numerous patterns observed in practice and a variety of possible individual behaviors.

At the individual scale, bacteria as E. coli or B. subtilis perform so-called run-and-tumble movements. This means that they alternate a run phase (or jump) followed by fast re-organization phase (tumble) in which they decide of a new direction for run. For this reason, the population is described by a nonlinear kinetic-Botlzmann equation of scattering type.

However, on-going experimental devices exhibit new behaviours and lead to new modeling questions. One can incorporate that the tumbling frequency is modulated by the intra-cellular state (receptor methylation level) and the extra-cellular environment (chemoattractant for instance). In the fast-adaptation and stiff response limit, we recover pathwise dependant tumbling frequencies. In the large scale limit (space, time) we can recover the Keller-Segel model.

This talk will present these aspects of the modeling, the connections between models through multiscale analysis, some qualitative behaviours and quantitative fitting with experiments. It is based on collaborations with biophysicists as well as mathematicians (N. Bournaveas, V. Calvez, M. Tang, N. Vauchelet).

Kui Ren

Statistical Physics of Large Dense Random Graphs

We study the asymptotics of large random graphs constrained by the limiting density of edges and the limiting subgraph density of an fixed graph H (for instance two stars and triangles etc). We show numerically and analytically (in some cases) that typical graphs with such constraints have very simple structures: asymptotically in the number of vertices there is a partition of the vertices into $M < \infty$ subsets $V_1, V_2, \ldots, V_M$, and a set of well-defined probabilities g_{ij} of an edge between any $v_i \in V_i$ and $v_j \in V_j$. We discuss briefly a possible application of this phenomenon study in image processing. This is a joint work with Richard Kenyon (Brown), Charles Radin (Austin) and Lorenzo Sadun (Austin).

Christian Ringhofer

A kinetic approach to behavioral game theory with applications to modeling insurance strategies

In this talk we discuss kinetic models for an ensemble of rational agents, trying to optimize an individual goal, given the behavior of the total ensemble. Agents have the ability to learn, i.e. they make decisions based on their past experience. The result is a system of kinetic equations for the distribution of strategy and 'experience'. We discuss in detail a particular application, namely the optimal design of insurance policies.

Yao Yao

Long time behavior of solutions to the 2D Keller-Segel equation with degenerate diffusion

Abstract: The Keller-Segel equation is a nonlocal PDE modeling the collective motion of cells attracted by a self-emitted chemical substance. When this equation is set up in 2D with a degenerate diffusion term, it is known that solutions exist globally in time, but their long-time behavior remain unclear. In a joint work with J.Carrillo, S.Hittmeir and B.Volzone, we prove that all stationary solutions must be radially symmetric up to a translation, and use this to show convergence towards the stationary solution as the time goes to infinity. I will also discuss another joint work with K.Craig and I.Kim, where we let the power of degenerate diffusion goes to infinity in the 2D Keller-Segel equation, so it becomes an aggregation equation with a constraint on the maximum density. We will show that if the initial data is a characteristic function, it will converge to the characteristic function of a disk as the time goes to infinity with certain convergence rate.