Abstracts

Tom Hou, Caltech

Intrinsic Sparse Mode Decomposition of High Dimensional Random Fields with Application to Stochastic Elliptic PDEs

Inspired by the recent developments in data sciences, we introduce an intrinsic sparse mode decomposition method for high dimensional random fields. This sparse representation of the random field allows us to break a high dimensional stochastic field into many spatially localized modes with low stochastic dimension. Such decomposition enables us to break the curse of dimensionality in our local solvers. To obtain such representation, we first decompose the covariance function into low rank part plus a sparse part. We then extract the spatially localized modes from the sparse part by minimizing a surrogate of the total area of the support. Moreover, we provide an efficient algorithm to solve it. As one example of application, we apply this technique to solve stochastic elliptic PDEs with high dimensional stochastic coefficients. Various combinations of local and global solvers achieve different level of accuracy and efficiency. Complexity and accuracy analysis will be demonstrated. At the end of the talk, I will also briefly discuss other applications of the intrinsic sparse mode extraction. This is a joint work with Qin Li and Pengchuan Zhang.

Bruno Despres, University of Paris VI

uncertainty propagation with intrusive kinetic methods for conservation laws

This presentatin will address the mathematical structure of intrusive (systems of) conservation laws, explain how to use the entropy method to generate hyperbolic intrusive systems (with G. Poette and D. Lucor) and show new results obtained recently with B. Perthame about the kinetic formulation of intrusive methods for uncertain conservation laws. In particular we will show how the new kinetic approach generates numerical methods with enhanced stability properties.

Jian-Guo Liu, Duke University

Stability of micro-macro decomposition base stochastic Galerkin method for linear transport equation with random imputs and diffusive scalings

We introduced a micro-macro decomposition based stochastic Galerkin method for multiscale linear transport equations with possibly diffusive scaling and random scattering coefficients. This method is made asymptotic-preserving, in the sense that it captures the diffusive limit without resolving the microscopic scales for time step, mesh size and modes in the polynomial chaos expansion. We give a stability result which shows that scheme is uniformly stable with respect to the mean free path. This is a joint work with Shi Jin.

Chi-Wang Shu, Brown University

High order DG and WENO methods for correlated random walk with *density-dependent turning rates*

We consider high order accurate approximations to the semilinear hyperbolic system of a correlated random walk model describing movement of animals and cells in biology. This system involves global integral source terms, making the design and analysis of stable schemes more complicated. We study both Runge-Kutta discontinuous Galerkin (RKDG) schemes, which are suitable for smooth solutions with the need for h-p adaptivity, and weighted essentially non-oscillatory (WENO) finite difference schemes, which are suitable when the solution contains discontinuities. Besides the standard L^2 stability and error estimates for the RKDG schemes, we also consider two different strategies to obtain positivity-preserving property without compromising accuracy, one for the RKDG schemes and one for the WENO finite difference schemes. Numerical experiments are performed to verify the good performance of the schemes. This is a joint work with Yan Jiang, Jianfang Lu and Mengping Zhang. [1] J. Lu, C.-W. Shu and M. Zhang, Stability analysis and a priori error estimate of explicit Runge-Kutta discontinuous Galerkin methods for correlated random walk with density-dependent turning rates, Science China Mathematics, 56 (2013), 2645-2676. [2] Y. Jiang, C.-W. Shu and M. Zhang, High order finite difference WENO schemes with positivity-preserving limiter for correlated random walk with density-dependent turning rates, Mathematical Models and Methods in Applied Sciences ($M^3 AS$), to appear.

Cory Hauck, Oak Ridge National Lab
Two-level sampling strategies for hyperbolic relaxation laws

We present work in progress on the development of two-level sampling strategies for collisional kinetic equations and hyperbolic relaxation laws. These strategies are based on a two-level approach, where one level comes from an asymptotic approximation around the relaxation limit and the second comes from the remaining deviation. The goal then is to balance stochastic and model errors in the two levels so that the overall cost with respect to sampling the original model is reduced.

Mohammad Motamed, New Mexico State University
Uncertainty quantification for high frequency waves

Eitan Tadmor, University of Maryland, College Park
Entropy stability and TeCNO computation of entropy measure-valued solutions

Entropy stability plays an important role in the dynamics of nonlinear systems of conservation laws. Entropic solutions need not be unique. Instead, they could be interpreted in an averaged sense as measure-valued solutions, averaged over the configuration space. We revisit the general framework of numerical entropy stability for difference approximations of such nonlinear equations. Our approach is based on comparing numerical viscosities with entropy conservative schemes. We demonstrate this approach with a host of high order entropic schemes. In particular, this paradigm serves as the building block for a class of non-oscillatory entropic schemes of arbitrarily high-order of accuracy, called TeCNO schemes. Numerical experiments provide a remarkable evidence for the effectiveness of the TeCNO schemes. These include recent TeCNO-based computation of entropy measure-valued solutions.

Christoph Schwab, ETH

Multilevel Monte-Carlo FV and FT Methods for hyperbolic PDEs with random input data

We consider random scalar, nonlinear hyperbolic conservation laws in spatial dimension d >=1 with bounded random flux functions. There exists a unique random entropy solution (i.e., a strongly measurable mapping from a probability space into C([0, T];L1(Rd))) with finite second moments. We present a convergence analysis of a Multi-Level Monte-Carlo Front-Tracking (MLMCFT) algorithm. It is based on “pathwise" application of the Front-Tracking Method for deterministic SCLs. We compare the MLMCFT algorithms to Multi-Level Monte-Carlo Finite-Volume methods. Due to the absence of a CFL time step restriction in the pathwise front tracking scheme, we can prove favourable complexity estimates: in spatial dimension d >= 2, the mean field of the random entropy solution can be approximated numerically with (up to logarithmic terms) the same complexity as the solution of one instance of the deterministic problem, on the same mesh. We then present results on large scale simulations of MLMC for linear acoustic wave propagation in heterogeneous media with log-gaussian random coefficients. Here, conventional explicit timestepping schemes encounter the CFL constraint which, due to the lognormal gaussian constitutive parameter, is random. A probabilistic complexity analysis is presented. Implementation with a novel adaptive load balancing algorithm achieves near linear strong scaling. Joint work with S. Mishra, N. Risebro, J. Sukys and F. Weber.

Siddhartha Mishra, ETH

UQ for hyperbolic conservation laws within the framework of measure valued solutions

Random entropy solutions, where the solutions and uncertain input are modeled with random fields, have been proposed as a framework for Uncertainty quantification for hyperbolic conservation laws in recent years. Wellposedness as well convergent numerical approximation schemes have been obtained for both scalar conservation laws as well as linear symmetrizable systems. We start by showing that the framework of random entropy solutions may not be adequate to describe UQ for nonlinear hyperbolic systems, particularly in several space dimensions. Moreover, we propose a different notion of solutions, that of measure valued solutions as an appropriate framework. Convergent numerical methods, of both the MonteCarlo as well as Multi-level MonteCarlo type are presented. We end with a short description of recent work on the computation of statistical solutions for turbulent flows.

Dongbin Xiu, University of Utah

Local polynomial chaos expansion for high dimensional stochastic PDE

We present a localized polynomial chaos expansion for PDE with random inputs. In particular, we focus on time independent linear stochastic problems with high dimensional random inputs, where the traditional polynomial chaos methods, and most of the existing methods, incur prohibitively high simulation cost. The local polynomial chaos method employs a domain decomposition technique to approximate the stochastic solution locally. In each subdomain, a subdomain problem is solved independently and more importantly, in a much lower dimensional random space. In a post-procesing stage, accurate samples of the original stochastic problems are obtained from the samples of the local solutions, by enforcing the correct stochastic structure of the random inputs and the coupling conditions at the interfaces of the subdomains. Overall, the method is able to solve stochastic PDEs in very large dimensions by solving a collection of low dimensional local problems and can be highly efficient. We present the mathematical framework of the method and use numerical examples to demonstrate its efficiency.

Heyrim Cho, Brown University
Uncertainty Quantification based on the joint response-excitation probability density and its application to stochastic Burgers equation

The joint response-excitation PDF approach enables us to obtain the probability density of the solution corresponding to a broad range of stochastic systems involving colored noise. We develop efficient numerical algorithms to solve this system from low- to high- dimensions. In addition, we employ dimension reduction techniques such as Mori-Zwanzig approach to obtain reduced order PDF equations. The effectiveness of our approach is demonstrated in various stochastic dynamical systems and stochastic PDEs. In particular, we study the stochastic Burgers equation with random initial states and random additive noise, yielding multiple interacting shock waves at random space-time locations.

Jingwei Hu, Purdue University
A stochastic Galerkin method for the nonlinear Boltzmann equation with uncertainty

We develop a stochastic Galerkin method for the nonlinear Boltzmann equation with uncertainty. The method is based on the generalized polynomial chaos and can handle random inputs from collision kernel, initial data or boundary data. We show that a simple SVD of gPC related coefficients combined with the Fourier-spectral method (in velocity space) allows one to compute the collision operator efficiently. Several numerical examples are presented to illustrate the validity of the proposed scheme. This is joint work with Shi Jin.

Max Gunzburger, Florida State University
Multilveve Monte Carlo and Stochastic Collocation Methods

We begin with a review of multilevel Monte Carlo (MLMC) methods for the solution of PDEs with random input data. We then consider stochastic collocation (SC) methods for this purpose. To alleviate the he curse of dimensionality, i.e., the explosive of the computational effort as the stochastic dimension increases, we propose and analyze a multilevel version of the SC method that, as is the case for MLMC methods, uses hierarchies of spatial approximations to reduce the overall computational complexity. In addition, our proposed approach utilizes, for approximation in stochastic space, a sequence of multi-dimensional interpolants of increasing fidelity which can then be used for approximating statistics of the solution as well as for building high-order surrogates featuring faster convergence rates. A rigorous convergence and computational cost analysis of the new multilevel stochastic collocation method is provided in the case of elliptic equations, demonstrating its advantages compared to standard single-level SC approximations as well as MLMC methods. Numerical results are provided that illustrate the theory and the effectiveness of the new multilevel method. (Joint work with Peter Jantsch, Aretha Teckentrup, Clayton Webster).

Tao Tang, Hong Kong Baptist University

Hermite spectral methods and discrete least square projection with random Evaluations

This talk is concerned with approximating multivariate functions in an unbounded domain by using a discrete least-squares projection with random point evaluations. Particular attention is given to functions with random Gaussian or gamma parameters. By using the Hermite/Laguerre functions and a proper mapping parameter, the stability can be significantly improved even if the number of design points scales linearly with the dimension of the approximation space.

Richard Dwight, Delft University of Technology

Predictive turbulence closures with Bayesian model-scenario averaging

The turbulence closure model is the dominant source of error in most Reynolds-Averaged Navier–Stokes simulations, yet no reliable estimators for this error component currently exist. Here we develop a stochastic, a posteriori error estimate, calibrated to specific classes of flow. It is based on variability in model closure coefficients across multiple flow scenarios, for multiple closure models. The variability is estimated using Bayesian calibration against experimental data for each scenario, and Bayesian Model-Scenario Averaging (BMSA) is used to collate the resulting posteriors, to obtain a stochastic estimate of a Quantity of Interest (QoI) in an unmeasured (prediction) scenario.

Tim Barth, NASA Ames

Combined Uncertainty and A-Posteriori Error Bound Estimates for General CFD Calculations

Hydrodynamic realizations often contain numerical error arising from finite-dimensional approximation (e.g. numerical methods using grids, basis functions, particles, etc) and statistical uncertainty arising from incomplete information and/or statistical characterization of model parameters and random fields. In this presentation, we posit that a general framework for uncertainty and error quanitification should include the combined effects of statistical uncertainty and numerical error so that uncertainty statistics with a-posteriori error bound estimates are provided. For problems containing no sources of uncertainty, a standard error bound estimate is obtained. For problems containing no numerical error, a standard uncertainty estimate is obtained. Specifically, we consider error bounds for moment statistics given realizations containing finite-dimensional numerical error (Barth, 2013). The error in computed output statistics contains contributions from both realization error and the error resulting from the calculation of statistics integrals using a numerical method. We then devise computable a-posteriori error bounds by numerically approximating all terms arising in the error bound estimates for a variety of standard UQ methods * Dense tensorization basis methods (Tatang,1994) and a subscale recovery variant (Barth,2013) for piecewise smooth data, * Sparse tensorization basis methods (Smolyak,1963) utilizing node-nested hierarchies, * Multi-level sampling methods (Mishra,2010) for high-dimensional random variable spaces. We have developed a general software package that provides the necessary tools and graphical user interface (GUI) for rapidly posing uncertainty quantification problems to a CFD method and calculating uncertainty statistics with error bounds for output quantities of interest. Example CFD calculations are shown to demonstrate features of the general framework. T.J. Barth,``Non-intrusive Uncertainty with Error Bounds for Conservation Laws Containing Discontinuities,'' ewblock Springer-Verlag Publishing, LNCSE, Vol 92, 2013. S. Smolyak,``Quadrature and Interpolation Formulas for Tensor Products of Centain Classes of Functions," Dok. Akad. Nauk SSSR, Vol. 4, 1963. M.A. Tatang,``Direct Incorporation of Uncertainty in Chemical and Environmental Engineering Systems,'" MIT, Dept. Chem. Engrg, 1994. S. Mishra and C. Schwab,``Sparse Tensor Multi-Level Monte Carlo Finite Volume Methods for Hyperbolic Conservation Laws,'', ETH Zurich, SAM Report 2010-24, 2010.

Clayton Webster, Oak Ridge National Lab

Quasi-optimal methods for deterministic and stochastic parameterized PDEs

In this talk, we present a generalized analytic framework for quasi-optimal polynomial and interpolation approximations, applicable to a wide class of parameterized PDEs with both deterministic and stochastic inputs. Such quasi-optimal methods construct an index set that corresponds to the “best M-terms,” based on sharp estimates of the polynomial coefficients. In particular, we consider several cases of N dimensional affine and non-affine coefficients, and prove analytic dependence of the PDE solution map in a polydisc or polyellipse of the multi-dimensional complex plane respectively. The framework we propose for analyzing asymptotic truncation errors of quasi-optimal methods is based on an extension of the underlying multi-index set into a continuous domain, and then an approximation of the cardinality (number of integer multi-indices) by its Lebesgue measure. Several types of isotropic and anisotropic (weighted) multi-index sets are explored, and rigorous proofs reveal sharp asymptotic error estimates in which we achieve sub-exponential convergence rates with respect to the total number of degrees of freedom. Through several theoretical examples, we explicitly derive the the rate constant and use the resulting sharp bounds to illustrate the effectiveness of our approach, as well as compare our rates of convergence with current published results. Finally, computational evidence complements the theory and shows the advantage of our generalized methodology compared to previously developed estimates.

Themistoklis Sapsis, MIT

Quantification and prediction of rare events in water waves

The scope of this work is the development, application, and demonstration of probabilistic methods for the quantification and prediction of extreme events occurring in complex nonlinear systems involving water waves and mechanical vibrations. Although rare these transitions can occur frequently enough so that they can be considered of critical importance. We are interested to address two specific topics related to rare events in complex dynamical systems: i) we want to be able to perform short term prediction given that we are able to measure specific quantities about the current system state (Rare Event Prediction Problem); and ii) we want to be able to quantify the probability of occurrence of a rare event for a given energetic regime of the system (Rare Event Quantification Problem). We first use a new adaptive reduction method to analytically quantify the role of spatial energy localization on the development of nonlinear instabilities and the subsequent formation of rare events in water waves. We then prove that these localized instabilities are triggered through the dispersive 'heat bath' of random waves that propagate in the nonlinear wave field. The interaction of uncertainty (induced through the dispersive wave mixing) and nonlinear instability defines a critical length-scale for the formation of rare events. To tackle the first problem we rely on this property and show that by merely tracking the energy of the wave field over this critical length-scale allows for the robust, inexpensive prediction?of the location of intense waves with a prediction window of 25 wave periods. ?For the second problem, we also utilize the nonlinear stability analysis to decompose the state space into regions where rare events is unlikely to occur and regions that lead with high probability to the occurrence of a rare event. The two regions are treated differently and the information of the two regimes is merged through a total probability argument, allowing for the efficient quantification of rare events in nonlinear water waves and subjected (to those) mechanical systems.