John von Neumann: The Early Years, The Years at Los Alamos and the Road to Computing*

By Peter Lax

My twin aims are to paint a picture of the fertility, power and sweep of von Neumann's mind, and to describe how his ideas and actions shaped the future. Today, nearly 60 years after his death, he looms larger than ever as the prophet of the age of computers.

Von Neumann was many things but primarily a mathematician. His genius was in his mathematics, and a mathematical way, coupled to uncommon common sense, pervaded his thinking about everything. Had von Neumann lived his normal span of years, he would have certainly been honored by the Abel in mathematics, a Nobel Prize in Economics, Computer Science and another one in Mathematics: these prizes do not yet exist, but they are bound to be established eventually. So we are talking about a triple Nobelist, possibly a 3½ fold one, if we take into account his contributions to the foundations of quantum mechanics. But I am getting ahead of my story.

The story starts, as always, with the birth of the hero, on December 28, 1903, in Budapest. He was the oldest of three boys of an upper middle class Jewish family; his father Max was a banker. The end of the XIXth and the beginning of the XXth century were heady times for Budapest, well chronicled by John Lukács, in his book, *Budapest 1900*. They were especially heady for mathematics and physics. Fejér, the Riesz brothers, Polya and Szegö, Haar, Polányi, von Kármán, Szilard, George Hevesi, Wigner, Teller, Dennis Gábor, George Békesy were all born within a 25 year period. The school system, reformed by von Kármán's father, was sensitive to unusual talent; so it is not surprising that Ráez László, teacher of mathematics at the Evangelical Gymnasium (among whose students more than 50% were Jewish), recognized immediately the Neumann boy's extraordinary gift. He informed Janesi's parents, and also Józef Kürschák, Nestor of the Hungarian mathematical community,

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and it was arranged that the young Neumann boy would receive special instruction. His first tutor was Gábor Szegö, himself a former prodigy, later a professor at Königsberg and then at Stanford; Mrs. Szegö liked to recall that her husband came home with tears in his eyes after his first encounter with the young boy genius. When Szegö left for Germany, Michael Fekete, later at the Hebrew University in Jerusalem, took over as tutor. Von Neumann's first publication was a joint paper with Fekete on the transfinite diameter, written in 1922 when von Neumann was 19; Fekete devoted the rest of his long scientific career to this subject.

Child prodigies are not rare in mathematics. The most likely reason, in addition to a brain wired for making logical connections, is that in order to grasp and solve mathematical problems no understanding is needed of wider context, which can be acquired only through worldly experience. This has, for many mathematicians, the unfortunate consequence that they shy away from mathematical problems posed in a non-mathematical setting. Not for all, to be sure; but few have embraced real world problems so wholeheartedly as von Neumann. According to his closest friend, the mathematician Stan Ulam, von Neumann's thinking was not geometric, nor tactile, but of an algebraic sort, games played with algebraic symbols on the one hand, and an interpretation of their meaning on the other. This may explain his ability to think in so many different millieus.

After finishing the gymnasium, his father decided that mathematics was an impractical career; chemical engineering was a much more promising profession. So young John went off, first to Berlin, and two years later to Zürich. There he made the acquaintance, or rather they of him, of two leading mathematicians, George Polya and Hermann Weyl, the latter one of the leaders of the intuitionists. In 1926 he received his degree in Zurich. But he also enrolled at the University of Budapest for a Ph.D. degree in mathematics, to be earned mostly in absentia. He was not yet 23 years old.

In Berlin von Neumann prepared for the entrance examination to the Federal Institute of Technology, which he passed "with outstanding result" in 1923; twenty years before, young Einstein failed. During the same period the young von Neumann* began to write his mathematical dissertation on the technical sounding by philosophically deep subject, "The Introduction of Transfinite Ordinal". It was eventually published under the title "An Axiomatisation of Set Theory." Its purpose was to resolve a slowly brewing crisis in mathematics: here is how von Neumann later described the problem.

> *In the late 19th and early 20th centuries a new branch of abstract mathematics, Georg Cantor's theory of sets, led into difficulties. That is, certain reasonings led to contradictions; and, while these reasonings were not in the central and useful part of set theory, and were always easy to spot by certain formal criteria, it was nevertheless not clear why they should be less legitimate than the successful parts of the theory*.

This split the mathematical community into two camps, the intuitionists, who would severely circumscribe the way infinite sets are handled, and the formalists, who believed that proper axiomatization, in the spirit of Euclid, would free us to handle infinite sets to our heart's desire, and at the same time deliver us from contradictions. The leader of the formalists was David Hilbert in Göttingen, the teacher of the leading mathematics Professor in Berlin, Erhardt Schmidt. Schmidt befriended the young von Neumann; many years later, in 1954, von Neumann expressed his gratitude by contributing to a Festschrift honoring the aged Schmidt, although by this time von Neumann had long ceased to bother about technical mathematics, and his numerous obligations ordinarily prevented him from writing papers.

His work on the foundations of set theory attracted the attention of the aging Hilbert in Göttingen, and his growing reputation earned him a grant from the Rockefeller Foundation to spend a year at Göttingen. When he arrived there he found that the burning issue of the day was not the foundation of set theory but the newly created quantum mechanics. The mathematics needed to clear up these new theories of Heisenberg and Schrödinger occupied von Neumann, on and off, the rest of his life. The theory he created of unbounded self-adjoint operators in

* When he moved to Germany, he started to use the appellation *von.* a loose translation of the title Margittai awarded to his father.

Hilbert space gives a logically satisfactory basis of quantum mechanics, and is a basic building stone of modern mathematics as well. Furthermore – and this was typical of von Neumann – he not only laid foundations but showed how they could be applied in specific, physically interesting situations.

By this time von Neumann's reputation was soaring. He was appointed Private Dozent in Berlin, and subsequently in Hamburg; he was invited to lecture all over Europe. But by the end of the twenties his eye was on America, partly because of the paucity of jobs in Europe, which he saw coming long before most other people did. So when an invitation for Princeton arrived in 1929 to lecture on mathematical physics, mainly the new quantum mechanics, he accepts with alacrity. For the next four years he divided his time equally between Princeton and Germany.

A scientifically important event for von Neumann in these years was Gödel's proof that the Hilbert program of formalism was doomed to failure. Gödel has shown in 1931 that a sufficiently rich logical system can never be proved to be free of contradiction, except by resorting to a richer logical system. This ended von Neumann's affair with axiomatics and set theory; yet his effort had not been in vain; they served him well thinking about the architecture of computers. A second event in 1932, decisive for the future, was Chadwick's discovery of neutrons.

The idyllic 50/50 arrangement came to an abrupt end in 1933 for two reasons; Hitler's rise to power, and von Neumann's appointment as professor at the newly created Institute for Advanced Study, also at Princeton. This was a very prestigious position; Albert Einstein and Hermann Weyl were fellow professors, and Gödel was to join later.

The mid-thirties were a fertile period for von Neumann. In collaborations with Francis Murray he made one of his most enduring discoveries, the theory of rings of operators, today called von Neumann algebras. At the same time the gathering political crises convinced him that war was inevitable, and that it would come soon. He also foresaw that it would lead to the destruction of the European Jews, much as the Armenians had been destroyed by the Turkish government during the first World War.

It is not surprising therefore that, feeling keenly that war was coming, he thought of ways to use his mathematical talents to help America prepare for war. The most mathematical part of warfare at the time was ballistics. The Aberdeen Proving Grounds were conveniently located not far from Princeton; so he threw himself energetically into the study of explosions and shock waves. In the process he almost became an army lieutenant in the Department of Ordnance, except that he was (barely) over the age limit of 35, and the Secretary of War would make no exceptions. This happily saved von Neumann from the shackles of the Army, so that he was able to roam freely over a wide range of projects. He was appointed to a number of committees, and actively participated in their deliberations. Soon his fame as a practical applied mathematician began to spread, just as his fame as a brilliant pure mathematician had spread fifteen years earlier. Among his new admirers were General Simon in the Department of Ordnance, and Vannevar Bush, head of the Office of Scientific Research and Development. Early in 1943 he was sent over to England to help out in antisubmarine and aerial warfare; he was able to help and in turn learned a great deal from the British about detonations. Soon he was able to put all his recently acquired knowledge to use for the important project of the ware, the making of an atomic – more precisely, nuclear – bomb.

At the time von Neumann arrived at Los Alamos there were many open problems, each of which had to be overcome in order to successfully construct a plutonium bomb. An isotope of plutonium fissions spontaneously and emits neutrons, in sufficient quantity to predetonate any bomb unless it was assembled fast enough. Implosion was the method most promising for assembly. Von Neumann's earlier knowledge of high explosives steered him to a safe and fast way to accomplish it. This and his many other technical contributions to solving physics and engineering problems established his reputation as the person to consult. He was admired by the brightest stars at Los Alamos: Oppenheimer, Bethe, Feynman, Peierls, Teller and many others; they acknowledged him as their superior for sheer brain power.

Nuclear weapons cannot be designed by trial and error; each proposed design has to be tested theoretically. This requires solving equations of compressible flows, governed by nonlinear equations.

Von Neumann came to the conclusion that analytical methods were inadequate for the task, and the only way to deal with equations of continuum mechanics is to discretize them and solve the resulting system of equations numerically. The tools needed to carry out such calculations effectively are high speed, programmable electronic computers, large capacity storage devices, programming languages, a theory of stable discretization of differential equations, and a variety of algorithms for solving rapidly the discretized equations. It is to these tasks that von Neumann devoted a large part of his energies during and after the war. He was keenly aware that computational methodology is crucial not only for designing weapons, but also for an enormous variety of scientific and engineering problems; understanding the weather and climate particularly intrigued him. But he also realized that computing can do more than grind out by brute force the answer to a concrete question.

I quote from a talk at Montreal delivered in 1945, when fast computers were merely figments of his imagination, he said: "We could, of course, continue to mention still other examples to justify our contention that many branches of both pure and applied mathematics are in great need of computing instruments to break the present stalemate created by the failure of the purely analytical approach to nonlinear problems. Really efficient high-speed computing devices may, in the field of nonlinear partial differential equations as well as in many other fields which are now difficult or entirely denied access, provide us with those heuristic hints which are needed in all parts of mathematics for genuine progress. In the specific case of fluid dynamics these hints have not been forthcoming for the last two generations from the pure intuition of mathematicians, although a great deal of first-class mathematical effort has been expended in attempts to break the deadlock in that field. To the extent to which hints arose at all, they originated in physical experimentation. We can now make computing so much more efficient, fast and flexible that it should be possible to use the new computer to supply the needed heuristic hints. This should ultimately lead to important analytic advances."

Everybody here knows that John von Neumann was the founding father of the modern computer; not everybody realizes that he was also the founding father of computational fluid dynamics. I will now closely describe 2 of his contributions to this subject.

One of von Neumann's fundamental contributions to the theory of difference equations was a notion of stability; an important test for it is named after him. As originally stated by him, this test implies the stability only of linear equations with constant coefficients; but von Neumann boldly asserted that it applies also to systems with variable coefficients – and so it turned out to be.

The deepest idea of von Neumann for computing compressible flow is shock capturing. This means that shocks and other discontinuities that inevitably arise in such flows are represented in the discrete approximations not as interior boundaries but as rapid transitions, and all points in the flow field are treated as ordinary points. In a calculation performed in 1944, von Neumann successfully studied the flow of gas in a tube one of whose ends in closed off; initially backwards, opposite to the flow direction. The paths of the particle change direction abruptly at the shock. Von Neumann observed that the particle paths are wobbly near the shock; this indicated that the velocity field is oscillating there. These oscillations are due to the dispersive nature of the difference equations which von Neumann employed. Subsequently, in a paper with Richtmyer, artificial viscosity was introduced that eliminated the unwanted oscillations.

If von Neumann were to awaken today, what would he find most astonishing? The tremendous power, low cost, and ubiquity of personal computer? The internet? Advances in computer and computation science? That the calculation of the climate still is a difficult problem? The decoding of the genome? The landing on the moon? The collapse of the Soviet Union? Or that the world has not blown itself up?

The tragedy of von Neumann's early death robbed mathematics and the sciences of a natural leader and an eloquent spokesman, and deprived a whole younger generation of beholding the most scintillating intellect of the twentieth century.