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Lions, Pierre-Louis (F-PARIS9-A); **Perthame, Benoît** (F-ORLN);
Tadmor, Eitan (IL-TLAV)

Formulation cinétique des lois de conservation scalaires multidimensionnelles. (French. English summary) [Kinetic formulation of multidimensional scalar conservation laws]

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This is a major paper concerning scalar conservation laws. Although the results hold independently on the space dimension N, they are new even if $N = 1$: a quantitative version of L. Tartar's compactness result [in *Nonlinear analysis and mechanics: Heriot-Watt Symposium, Vol. IV*, 136–212, Pitman, Boston, MA, 1979; MR0584398 (81m:35014)] is given.

Thanks to a forthcoming paper of two of the authors, the equation $\partial_t u + \operatorname{div} A(u) = 0$, $t > 0$, is reformulated as a kinetic equation. Its well-posedness, regarding the Cauchy problem, is proved.

Let us assume that $\operatorname{meas}\{-R < v < R;\ \tau + A'(v)\cdot\xi = 0\} = 0$ for any R. Then any bounded sequence of solutions, bounded in mass, is actually relatively compact in $L^1_{\rm loc}$. If $N = 1$, this is Tartar's result, otherwise it is new.

More precisely, one may assume that

$$\operatorname{meas}\{-R < v < R;\ |\tau + A'(v)\cdot\xi|^2 \le \delta^2(\tau^2 + |\xi|^2)\} \le C(R)\delta^\alpha,$$

uniformly in (τ, ξ). Then such a sequence of solutions is actually bounded in $W^{s,p}_{\rm loc}$, uniformly in time, for $s < \alpha/(\alpha+2)$ and $p = (\alpha+4)(\alpha+2)$. Examples indicate that the bound for s is not accurate; α would be expected.

Reviewed by *Denis Serre*