ERRATUM: EULERIAN DYNAMICS WITH A COMMUTATOR FORCING

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The publication [1] has a minor gap in the argument presented in Section 6.2 where the authors establish control over the first derivatives of density and momentum. Specifically, the bound on $\Lambda\rho$ used in the momentum equation involves term $\sqrt{D\rho'(x)}$, which propagates into formula (6.21). At that point the authors combined (6.21) with (6.19) to get rid of the D-term. The mistakes presents in the fact that the point x at which the D-term is evaluated in 6.19 is different from the point x at which it is evaluated in 6.21. Hence the values may be different.

To avoid using combination of 6.19 and 6.21 we argue as follows. We produce a uniform bound on $|\rho''|_2$ on the time interval in question. This uniform bound, by Sobolev embedding, implies that $\rho' \in C^{\frac{1}{2}}$ uniformly. Then the trivial bound

$$|\Lambda\rho|_\infty \leqslant |\rho'|_{C^{1/2}},$$

implies uniform control over $\Lambda\rho$. Hence it is not necessary to resort to 6.19 to contain $\Lambda\rho$, and the rest of the estimates on m' follow as documented in [1].

To achieve uniform bound on $|\rho''|_2$ we differentiate the density equation twice:

$$\partial_t\rho'' + u\rho''' + u'\rho'' + e''\rho + 3e'\rho' + 2e\rho'' = -2\rho''\Lambda\rho - 3\rho'\Lambda\rho' - \rho\Lambda\rho''.$$

Using that $u' = e + \Lambda\rho$, we obtain

$$\partial_t\rho'' + u\rho''' + e''\rho + 3e'\rho' + 3e\rho'' = -3\rho''\Lambda\rho - 3\rho'\Lambda\rho' - \rho\Lambda\rho''.$$

At this point we know that $|e^{(k)}| \lesssim \rho^{(k)}$, and we have uniform bounds on ρ, ρ'. So, testing with ρ'', integrating by parts in $u\rho'''\rho''$ term, and using the e quantity again, we obtain

$$\partial_t|\rho''|_2^2 \lesssim |\rho''|_2 + |\rho''|_2^2 + |\Lambda\rho|_\infty|\rho''|_2^2 + |\rho''|_2|\Lambda\rho'|_2 - \int_{\mathbb{T}} \rho\rho''\Lambda\rho'' dx.$$

Using that $|\Lambda\rho'|_2 \lesssim |\rho''|_2$, and log-Sobolev inequality

$$|\Lambda\rho|_\infty \leqslant |\rho'|_\infty(1 + \log_+|\rho''|_2) \lesssim 1 + \log_+|\rho''|_2,$$

we further obtain

$$\partial_t|\rho''|_2^2 \lesssim C + |\rho''|_2^2(1 + \log_+|\rho''|_2) - \int_{\mathbb{T}} \rho\rho''\Lambda\rho'' dx.$$

Using symmetrization in the remaining dissipation term we have

$$-\int_{\mathbb{T}} \rho\rho''\Lambda\rho'' dx = -\int_{\mathbb{T}} \rho D\rho'' dx + R, \tag{0.1}$$

where

$$R = \int_{\mathbb{T}} \rho''(x) \int_{\mathbb{T}} \frac{(\rho(x) - \rho(y))(\rho''(x) - \rho''(y))}{|x-y|^2} dy dx.$$

Date: October 21, 2018.

Using the bound $|\rho'| < C$ we further conclude

$$|R| \lesssim \int_{\mathbb{T}} |\rho''(x)| \int_{\mathbb{T}} \frac{|\rho''(x) - \rho''(y)|}{|x-y|} dydx \leqslant \int_{\mathbb{T}} |\rho''(x)| \sqrt{D\rho''} dx \leqslant |\rho''|_2 \sqrt{\int_{\mathbb{T}} D\rho'' dx}.$$

By Young, the latter is bounded by

$$|R| \leqslant \varepsilon \int_{\mathbb{T}} D\rho'' dx + C_\varepsilon |\rho''|_2^2,$$

where ε is smaller than the lower bound on the density on the given time interval. This gets the D-term absorbed into dissipation term in (0.1). We thus arrive at

$$\partial_t |\rho''|_2^2 \lesssim C + |\rho''|_2^2 (1 + \log_+ |\rho''|_2).$$

The result follows by integration.

Since in the estimates above we relied on second order a priori bound $|e''| \lesssim |\rho''|$ it is necessary to raise the regularity class from H^3 as in [1] to H^4 so that the local transport equation for e'' can be solved classically. The idea to avoid using higher order a priori bounds $|e^{(k)}| \lesssim |\rho^{(k)}|$ is to abandon the use of momentum equation for m, where e quantity is explicitly present, and instead come back to the u-equation. This was performed in [1] up to the order 3 space H^4, and the argument is entirely similar going one more derivative up to H^4. We therefore state our final result as follows.

Theorem 0.1. *Consider the system of equations (1.1), [1], with $1 \leqslant \alpha < 2$ subject to initial data $(u_0, \rho_0) \in H^4(\mathbb{T}^1) \times H^{3+\alpha}(\mathbb{T}^1)$. Then the system admits a global solution in the same class.*

References

[1] R. Shvydkoy and E. Tadmor, *Eulerian dynamics with a commutator forcing*, Trans. Math. and Appl. 1(1) (2017) 1-26.

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