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Enhanced Edge Detection Method Based on a Threshold Function for Discrete Data

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Abstract

In this paper we develop enhanced edge detectors based on existing local edge detectors and a threshold function composed of the sigmoidal transformation. It is proved that the proposed edge detectors can eliminate inevitable oscillations of the original edge detectors near jump discontinuities and improve the resolution far from the discontinuities. Furthermore, we suggest a scheme for applying the proposed method to the two dimensional discrete data. We include some examples to show that the numerical results are consistent with the theoretical analysis of the proposed edge detectors.

Mathematics Subject Classification: 65D18, 68W25

Keywords: edge detector; jump discontinuity, sigmoidal transformation, threshold function

1 Introduction

Accurate edge detection is essential, for example, in image processing and pattern recognition because the most significant information is often observed near the edges which are identified by jump discontinuities in a given data. Until now many edge detection methods have been introduced in the literature [1–4, 6–14] which are classified two categories, the gradient based methods and the zero-crossing based methods, in general.

In practice, jump discontinuities occur at every grid point in given discrete data, and thus determining existence of the jump discontinuities which is identified with the edges is not a simple problem. Gelb and Tadmor [6–9] proposed a family of edge detectors accompanying concentration factors near the jump discontinuities of the discrete data. Convergence rate of higher order edge detectors is faster than lower ones away from jump discontinuities while the higher order ones generate annoying oscillations near the discontinuities. In addition, the edge detection method requires an outside threshold parameter which indicates the minimal magnitude below which jump discontinuities are neglected. Recently, in order to resolve the aforementioned problems of the existing edge detectors the author provided an adaptive edge detection method based on a nonlinear transformation in [15].

In this paper, extending the idea in the literature [15], we aim to develop an enhanced edge detection method for removing the unwanted oscillations near the jump discontinuities and improving resolution away from the discontinuities. In Section 2, for understanding of the motivations of the present work, Gelb and Tadmor's local edge detectors and sigmoidal transformations are introduced. In Section 3 we propose simple form of threshold functions associated with the sigmoidal transformation. Then the edge detectors and the proposed threshold function are combined to pinpoint significant edges of the given data as well as improve convergence far from the edges. Furthermore, the application to two dimensional data is given in Section 4. Numerical examples are included to demonstrate efficiency of the presented method.

2 Preliminaries

In this section we recall the edge detectors proposed by Gelb and Tadmor [6–9] and the sigmoidal transformations. For a $(b-a)$ periodic piecewise smooth function $f(x)$, $a \le x < b$, we define a jump function $[f](x) := f(x+) - f(x-)$, where $f(x\pm)$ indicate the right and left side limits of $f(x)$ at x. Let the data, $f_k = f(x_k)$ are given for $2N+1$ points $x_k = a + h \cdot k\,(k = 0, 1, \cdots, 2N)$ with $h = (b-a)/(2N+1)$. A point ξ such that $[f](\xi) \neq 0$ is called a jump discontinuity which we wish to identify. Edges of piecewise smooth data can be identified by jump discontinuities in the data.

In the literature [9] local edge detectors based on the difference formula were introduced as

$$g_{2p+1,h}(x) := \binom{2p}{p}^{-1} \Delta^{2p+1} f_j\,, \qquad x_j \le x < x_{j+1} \tag{1}$$

for each j, where $\Delta^{2p+1} f_j$ indicates the difference formula of order $2p+1$ for an integer $p \ge 0$. The local edge detector $g_{2p+1,h}(x)$ has a concentration property for a jump discontinuity ξ as follows.

$$g_{2p+1,h}(x) = \begin{cases} (-1)^l \frac{Q_{l,p}}{Q_{0,p}} [f](\xi) + O(h), & \text{if } x_{j-l} \le \xi < x_{j+1-l}, \quad |l| \le p \\ O(h^{2p+1}), & \text{otherwise} \end{cases} \tag{2}$$

as $h \to 0$, where $Q_{l,p} = \binom{2p}{p+|l|}$. This formula implies that the oscillatory behavior of $g_{2p+1,h}(x)$ near the discontinuity ξ is increasing as p grows.

In addition, we introduce a sigmoidal transformation $\gamma_m(t)$, $0 \le t \le 1$, of order m which has the so-called clustering property:

$$\gamma_m(t) = \begin{cases} O(t^m), & t < \frac{1}{2} \\ 1 + O((1-t)^m), & t > \frac{1}{2} \end{cases} \tag{3}$$

as m goes to the infinity. Definitions, classification with examples and general properties of the sigmoidal transformations are summarized in the literature [5]. On the other hand, as a particular type of the sigmoidal transformation, we recall the following sigmoidal transformation of infinity order.

$$\gamma_\infty(t) = \frac{1}{2} + \frac{1}{2}\tanh\left[\frac{r}{4}\left(\frac{1}{1-t} - \frac{1}{t}\right)\right] \tag{4}$$

with $r > \sqrt{3}$, which satisfies an additional property as

$$\gamma_\infty^{(j)}(0) = \gamma_\infty^{(j)}(1) = 0 \tag{5}$$

for all integers $j \ge 1$. In the numerical implementation later, to avoid the ambiguity of choosing the order m of $\gamma_m(t)$ we employ the infinity order sigmoidal transformation $\gamma_\infty(t)$.

3 Construction of enhanced edge detectors

In order to improve the performance of the existing local edge detectors $g_{2p+1,h}(x)$ in (1), referring to the literature [15], we define a function as follows: For an integer $p \ge 0$,

$$\theta_{2p+1}(x) = \frac{|g_{2p+1,h}(x)|}{|g_{2p+1,h}(x)| + \beta_{2p+1}\{|g_{2p+1,h}(x-h)| + |g_{2p+1,h}(x+h)|\} + \delta_{2p+1}} \tag{6}$$

with the parameters $\beta_{2p+1} > 0$ and $\delta_{2p+1} > 0$. We call $\theta_{2p+1}(x)$ a *threshold function* of order $2p+1$. A range of the parameter β_{2p+1} for each $p \ge 1$ is determined by the concentration property of $g_{2p+1,h}(x)$ and the clustering property of $\gamma_m(x)$ as shown below while β_1 may be selected arbitrarily. Then we propose a local edge detector of order $2p+1$ based on $\theta_{2p+1}(x)$ as

$$G^{[m]}_{2p+1,h}(x) = g_{2p+1,h}(x)\gamma_m\left(\theta_{2p+1}(x)\right), \qquad x_j \le x < x_{j+1}, \tag{7}$$

where γ_m is the sigmoidal transformation of order m introduced in the previous section.

For practical purpose, from now on, we will explore the cases of $p = 0$, 1, 2 successively. First, we consider the simplest local edge detector $g_{1,h}(x) = f_{j+1} - f_j$, $x_j \le x < x_{j+1}$, for each $j = 0, 1, \cdots, 2N$ and the first order threshold function

$$\theta_1(x) = \frac{|g_{1,h}(x)|}{|g_{1,h}(x)| + \beta_1\left\{|g_{1,h}(x-h)| + |g_{1,h}(x+h)|\right\} + \delta_1}. \tag{8}$$

From (2) we have a so-called concentration property

$$g_{1,h}(x) = \begin{cases} [f](\xi) + O(h), & \text{if } \xi \in [x_j,\, x_{j+1}) \\ O(h), & \text{otherwise} \end{cases} \tag{9}$$

as $h \to 0$ (or $N \to \infty$). Suppose that $\xi \in [x_j, x_{j+1})$. Then the property (9) results in

$$\theta_1(x) = \frac{|[f](\xi)|}{|[f](\xi)| + \delta_1}, \qquad x_j \le x < x_{j+1}$$

for sufficiently small $h > 0$, and thus

$$\theta_1(x) \gtrless \frac{1}{2}, \qquad \text{if } |[f](\xi)| \gtrless \delta_1. \tag{10}$$

On the other hand, when $\xi \notin [x_j, x_{j+1})$,

$$\theta_1(x) = O(h). \tag{11}$$

From the definition of $G^{[m]}_{1,h}(x)$ in (7) and the clustering property of $\gamma_m(x)$ in (3) it follows that for $x_j \le x < x_{j+1}$

$$G^{[m]}_{1,h}(x) = \begin{cases} [f](\xi) + O\left((1-\theta_1(x))^m\right), & \text{if } \xi \in [x_j,\, x_{j+1}) \text{ and } |[f](\xi)| > \delta_1 \\ O(\theta_1(x)^m), & \text{if } \xi \in [x_j,\, x_{j+1}) \text{ and } |[f](\xi)| < \delta_1 \\ O(h^{m+1}), & \text{otherwise} \end{cases} \tag{12}$$

as $h \to 0$. Comparing this result with the asymptotic behavior in (9), one can observe the followings:

(a) When $\xi \in [x_j,\, x_{j+1})$ and $|[f](\xi)| > \delta_1$, convergence rate of $G^{[m]}_{1,h}(x)$ to $[f](\xi)$ becomes higher as the amplitude of the discontinuity, $[f](\xi)$ is larger.

(b) Every jump discontinuity, with an amplitude below the threshold parameter δ_1, vanishes by $G^{[m]}_{1,h}(x)$ such as $O(\theta_1(x)^m)$.

(c) For sufficiently large m, $G^{[m]}_{1,h}(x)$ will improve convergence rate of $g_{1,h}(x)$ away from the jump discontinuity. Therefore it can be used to enhance separation of the jump discontinuity from smooth regions, which generates higher resolution in the practical edge detection.

(d) All the remarks (a), (b) and (c) hold regardless of the value of the parameter β_1.

For higher order local edge detectors we consider the third and fifth order difference formulas as

$$g_{3,h}(x) = \frac{1}{2}\{-f_{j-1} + 3f_j - 3f_{j+1} + f_{j+2}\} \tag{13}$$

and

$$g_{5,h}(x) = \frac{1}{6}\{-f_{j-2} + 5f_{j-1} - 10f_j + 10f_{j+1} - 5f_{j+2} + f_{j+3}\} \tag{14}$$

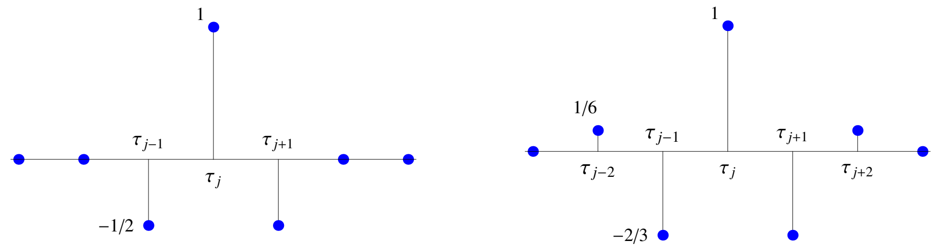
for all $x_j \le x < x_{j+1}$, $j = 0, 1, \cdots, 2N$. The concentration property in (2) implies

$$g_{3,h}(x) = \begin{cases} [f](\xi) + O(h)\,, & \text{if } \xi \in [x_j,\, x_{j+1}) \\ -\frac{1}{2}[f](\xi) + O(h)\,, & \text{if } \xi \in [x_{j-1},\, x_j) \cup [x_{j+1},\, x_{j+2}) \\ O(h^3)\,, & \text{otherwise} \end{cases} \tag{15}$$

and

$$g_{5,h}(x) = \begin{cases} [f](\xi) + O(h)\,, & \text{if } \xi \in [x_j,\, x_{j+1}) \\ -\frac{2}{3}[f](\xi) + O(h)\,, & \text{if } \xi \in [x_{j-1},\, x_j) \cup [x_{j+1},\, x_{j+2}) \\ \frac{1}{6}[f](\xi) + O(h)\,, & \text{if } \xi \in [x_{j-2},\, x_{j-1}) \cup [x_{j+2},\, x_{j+3}) \\ O(h^5)\,, & \text{otherwise.} \end{cases} \tag{16}$$

When a jump discontinuity ξ is included in $[x_j, x_{j+1})$, the concentration property of the edge detectors $g_{3,h}(x)$ and $g_{5,h}(x)$ for intermediate points $\tau_k = (x_k + x_{k+1})/2$, $k = j, j \pm 1, j \pm 2, j \pm 3$, are depicted in Fig 1. Therein, the heights indicate normalized values, $g_{q,h}(\xi)/[f](\xi)$ for each $q = 3, 5$. It should



(a) $g_{3,h}(x)$ (b) $g_{5,h}(x)$

Fig. 1. Concentration property of the edge detectors $g_{3,h}(x)$ and $g_{5,h}(x)$ when a jump discontinuity $\xi \in [x_j, x_{j+1})$ for intermediate points $\tau_k = (x_k + x_{k+1})/2$, $k = j, j \pm 1, j \pm 2, j \pm 3$.

be noted that the higher order edge detectors lead to faster convergence than the first order edge detector away from the the jump discontinuity while they generate oscillations near the discontinuity. Thus, in this case we aim to develop enhanced edge detectors which eliminate the oscillations produced by $g_{3,h}(x)$ and $g_{5,h}(x)$ near the jump discontinuities as well as further improve convergence far away.

Similarly to the case of $g_{1,h}(x)$, for m assumed to be large enough we define new higher order local edge detectors as follows.

$$G_{q,h}^{[m]}(x) := g_{q,h}(x)\gamma_m\left(\theta_q(x)\right), \tag{17}$$

where $q = 3$ or 5. The function $\theta_q(x)$ is defined as

$$\theta_q(x) = \frac{|g_{q,h}(x)|}{|g_{q,h}(x)| + \beta_q\left\{|g_{q,h}(x-h)| + |g_{q,h}(x+h)|\right\} + \delta_q}, \tag{18}$$

where $\delta_q > 0$ is a given small number and $\beta_q > 0$ is a parameter to be determined. Referring to the clustering property of the sigmoidal transformation $\gamma_m(t)$, we suggest the following conditions in order that $G_{q,h}^{[m]}(x)$ in (17) eliminates the oscillations of $g_{q,h}(x)$ near the jump discontinuity and improves convergence away from the discontinuity.

$$\theta_q(x) > \frac{1}{2}, \qquad \text{if } \xi \in [x_j, x_{j+1}) \tag{19}$$

and

$$\theta_q(x) < \frac{1}{2}, \qquad \text{if } \xi \notin [x_j, x_{j+1}) \tag{20}$$

for all $x_j \le x < x_{j+1}$. We assume that h is sufficiently small. Then for the case of $q = 3$ the concentration property in (15) and the conditions (19) and (20) imply that

$$\theta_3(x) = \frac{1}{1+\beta_3+\delta'} > \frac{1}{2}$$

if $\xi \in [x_j, x_{j+1})$, and

$$\theta_3(x) = \frac{1}{1+2\beta_3+2\delta'} < \frac{1}{2}$$

if $\xi \in [x_{j-1}, x_j) \cup [x_{j+1}, x_{j+2})$, in which $\delta' = \delta_3/|[f](\xi)|$ with a small $\delta_3 > 0$. Thus we have the range of the parameter β_3 as follows.

$$\frac{1}{2} - \delta' < \beta_3 < 1 - \delta'. \tag{21}$$

It should be noted that the parameter δ_3(and δ_5) in (18) exists only to prevent the denominator vanishing while δ_1 in (8) plays a role of a threshold of a jump discontinuity.

For the case of $q = 5$ the concentration property in (16) and the conditions (19) and (20) imply that

$$\theta_5(x) = \frac{1}{1+\frac{4}{3}\beta_5+\delta'} > \frac{1}{2}$$

if $\xi \in [x_j, x_{j+1})$,

$$\theta_5(x) = \frac{\frac{2}{3}}{\frac{2}{3}+\frac{7}{6}\beta_5+\delta'} < \frac{1}{2}$$

if $\xi \in [x_{j-1}, x_j) \cup [x_{j+1}, x_{j+2})$, and

$$\theta_5(x) = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{2}{3}\beta_5+\delta'} < \frac{1}{2}$$

if $\xi \in [x_{j-2}, x_{j-1}) \cup [x_{j+2}, x_{j+3})$. Thus for $\delta' = \delta_5/|[f](\xi)|$ with a small $\delta_5 > 0$ we have

$$\frac{4}{7}\left(1 - \frac{3}{2}\delta'\right) < \beta_5 < \frac{3}{4}\left(1 - \delta'\right). \tag{22}$$

From the definition of $G_{q,h}^{[m]}(x)$ in (17) and the clustering property (3) of $\gamma_m(x)$ we can see that the proposed edge detectors $G_{3,h}^{[m]}(x)$ with β_3 in (21) and $G_{5,h}^{[m]}(x)$ with β_5 in (22) have the following asymptotic equations: For $x_j \le x < x_{j+1}$

$$G_{3,h}^{[m]}(x) = \begin{cases} [f](\xi) + O((1-\theta_3(x))^m), & \text{if } \xi \in [x_j, x_{j+1}) \\ O(\theta_3(x)^m), & \text{if } \xi \in [x_{j-1}, x_j) \cup [x_{j+1}, x_{j+2}) \\ O(h^{3+3m}), & \text{otherwise} \end{cases} \tag{23}$$

and

$$G_{5,h}^{[m]}(x) = \begin{cases} [f](\xi) + O((1-\theta_5(x))^m)\,, & \text{if } \xi \in [x_j,\, x_{j+1}) \\ O(\theta_5(x)^m)\,, & \text{if } \xi \in [x_{j-2},\, x_j) \cup [x_{j+1},\, x_{j+3}) \\ O(h^{5+5m})\,, & \text{otherwise} \end{cases} \tag{24}$$

as $h \to 0$. The results in (23) and (24) indicate that $G_{q,h}^{[m]}(x)$ with m large enough can diminish the oscillations near the jump discontinuity as well as highly improve the rate of convergence away from the discontinuity. In other words, separation of the neighborhood of the jump discontinuity ξ from the smooth regions can be sufficiently enhanced by $G_{q,h}^{[m]}(x)$. On the other hand, unlike the case of the first order edge detector $G_{1,h}^{[m]}(x)$, the parameter δ_q in (18) does not play a role of the threshold because the conditions in (19) and (20) are satisfied regardless of the value of the magnitude $[f](\xi)$ of the jump discontinuity.

In order to avoid the problem of choosing an appropriate order m of the sigmoidal transformation, we may employ the infinite order sigmoidal transformation $\gamma_\infty(x)$ instead of $\gamma_m(x)$ in (7). This results in the local edge detector of infinite order

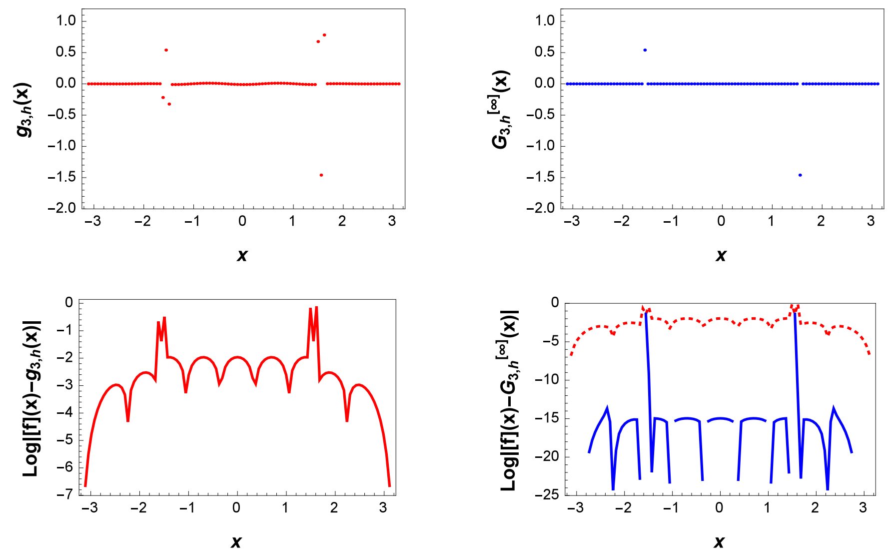
$$G_{2p+1,h}^{[\infty]}(x) = g_{2p+1,h}(x)\gamma_\infty\left(\theta_{2p+1}(x)\right) \tag{25}$$

For numerical experiment we take the following example used in the literature [9, 15].

$$f_1(x) = \begin{cases} \sin^7(x+\pi), & -\pi \le x < -\frac{\pi}{2} \\ \left(\dfrac{x}{\pi}\right)^3 - \sin\left(\frac{9x}{2}\right) + 1, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \sin^7(x-\pi), & \frac{\pi}{2} < x < \pi \end{cases} \tag{26}$$

which has jump discontinuities at $x = \pm\frac{\pi}{2}$.

Fig. 2 shows that the presented edge detector $G_{3,h}^{[\infty]}(x)$ with $N = 50$ enhances separation of the jump discontinuities from smooth regions. Moreover, log errors of $g_{3,h}(x)$ and $G_{3,h}^{[\infty]}(x)$ for the jump function $[f](x)$ are illustrated, which implies that $G_{3,h}^{[\infty]}(x)$ converges faster away from the discontinuities. In Fig. 2(b) the log error of $g_{3,h}(x)$ indicated by dotted line is included for comparison with that of $G_{3,h}^{[\infty]}(x)$. Therein, we have taken $\beta_3 = 0.75$ and $\delta_3 = 10^{-6}$ in (18). The result is consistent with the asymptotic behavior (23) of $G_{3,h}^{[m]}(x)$ with m large enough.



(a) $g_{3,h}(x)$ (b) $G^{[\infty]}_{3,h}(x)$

Fig. 2. Graphs of the third order edge detector $g_{3,h}(x)$ and its log error (in (a)) compared with those of the presented edge detector $G^{[\infty]}_{3,h}(x)$ (in (b)) for the two dimensional data of $f(x) = f_1(x)$ with $N = 50$.

4 Applications to two dimensional data

For simplicity we suppose that two dimensional data $f_{i,j} = f(x_i, y_j)$ are given at the grid points (x_i, y_j) in a rectangular region $[a, b) \times [c, d)$, where $x_i = a+h_1 \cdot i\,(i = 0, 1, \cdots, 2N)$ with $h_1 = \frac{b-a}{2N+1}$ and $y_j = c+h_2 \cdot j\,(j = 0, 1, \cdots, 2M)$ with $h_2 = \frac{d-c}{2M+1}$. In this case we define two dimensional local edge detector as

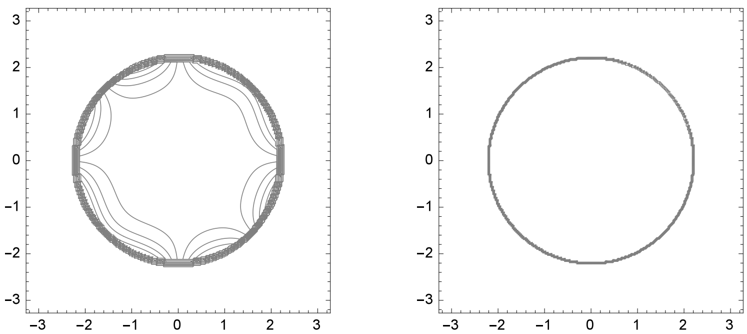
$$G^{[\infty]}_{q,h}(x, y) := G^{[\infty]}_{q,h_1}(x) \;+\; G^{[\infty]}_{q,h_2}(y), \tag{27}$$

for $x_i \le x < x_{i+1}$ and $y_j \le y < y_{j+1}$. Furthermore, we transform the range of the detectors onto the interval $[0, 1]$ as follows.

$$\widetilde{G}^{[\infty]}_{q,h}(x, y) := \frac{G^{[\infty]}_{q,h}(x, y) - \min\limits_{i,j}\left\{G^{[\infty]}_{q,h}(x_i, y_j)\right\}}{\max\limits_{i,j}\left\{G^{[\infty]}_{q,h}(x_i, y_j)\right\} - \min\limits_{i,j}\left\{G^{[\infty]}_{q,h}(x_i, y_j)\right\}}. \tag{28}$$

Similarly, we set $g_{q,h}(x, y)$ as

$$g_{q,h}(x, y) := g_{q,h_1}(x) \;+\; g_{q,h_2}(y) \tag{29}$$



(a) $\widetilde{g}_{5,h}(x,y)$ (b) $\widetilde{G}^{[\infty]}_{5,h}(x,y)$

Fig. 3. Contour plots of the edge detectors $\widetilde{g}_{5,h}(x,y)$ and $\widetilde{G}^{[\infty]}_{5,h}(x,y)$ for the two dimensional data of $f(x,y)=f_2(x,y)$ with $N=M=120$.

where, for example, for $q=5$

$$g_{5,h_1}(x)=\frac{1}{6}\left\{-f_{i-2,j}+5f_{i-1,j}-10f_{i,j}+10f_{i+1,j}-5f_{i+2,j}+f_{i+3,j}\right\}$$

and

$$g_{5,h_2}(y)=\frac{1}{6}\left\{-f_{i,j-2}+5f_{i,j-1}-10f_{i,j}+10f_{i,j+1}-5f_{i,j+2}+f_{i,j+3}\right\}.$$

In addition, we set the transformed edge detector $\widetilde{g}_{q,h}(x,y)$ like $\widetilde{G}^{[\infty]}_{q,h}(x,y)$ in (28).

For numerical experiment we take the following example given in [8].

$$f_2(x,y)=\begin{cases}3\cos\left(\frac{xy}{\pi}\right)-\sin\left(\frac{x}{2}\right)-\sin\left(\frac{y}{2}\right), & \text{if } x^2+y^2<(0.7\pi)^2\\ 0, & \text{otherwise}\end{cases}\tag{30}$$

on the region $-\pi\le x<\pi$, $-\pi\le y<\pi$. For the data of $f_2(x,y)$ with $N=M=120$, Fig. 3 compares contour plots of the edge detectors $\widetilde{g}_{5,h}(x,y)$ and $\widetilde{G}^{[\infty]}_{5,h}(x,y)$ with $\beta_5=0.7$ and $\delta_5=10^{-6}$ in (18). One can see that the presented edge detector $\widetilde{G}^{[\infty]}_{5,h}(x,y)$ provides clearer edges of $f_2(x,y)$ than $\widetilde{g}_{5,h}(x,y)$.

Additionally, we consider an example of a 223×227 jpeg image shown in Fig. 4. The contour plots of the existing local edge detector $\widetilde{g}_{5,h}(x,y)$ and the presented detector $\widetilde{G}^{[\infty]}_{5,h}(x,y)$ with $\beta_5=0.7$ and $\delta=10^{-6}$ are given in Fig. 5. The result of $\widetilde{g}_{5,h}(x,y)$ reveals unclear edges as well as some unwanted noises while $\widetilde{G}^{[\infty]}_{5,h}(x,y)$ shows highly improved resolution. The reason of the

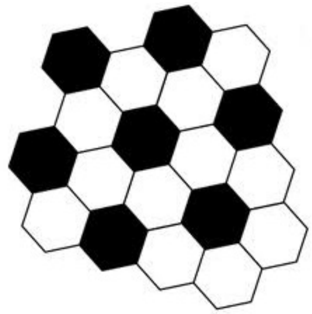
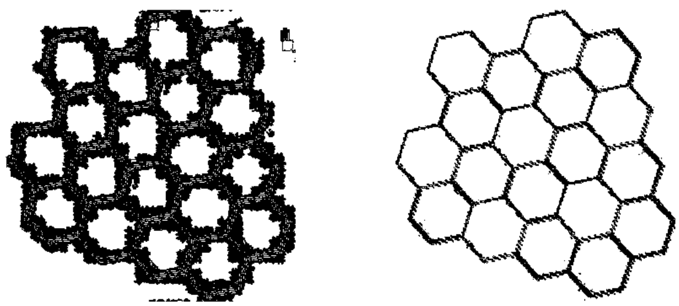


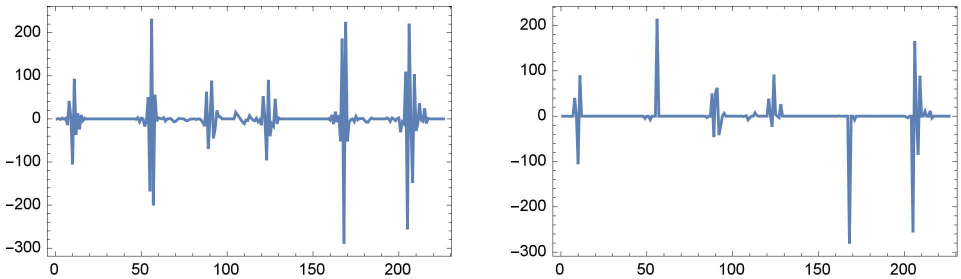
Fig. 4. An example of a 223×227 jpeg image.



(a) $\widetilde{g}_{5,h}(x,y)$ (b) $\widetilde{G}^{[\infty]}_{5,h}(x,y)$

Fig. 5. Contour plots of the edge detectors $\widetilde{g}_{5,h}(x,y)$ and $\widetilde{G}^{[\infty]}_{5,h}(x,y)$ for the image in Fig. 4.

resolution enhancement is that lots of the spurious oscillations of $g_{5,h}(x,y)$ near the edges are removed by $G^{[\infty]}_{5,h}(x,y)$ as illustrated in Fig. 6, where the graphs of $g_{5,h}(x,y)$ and $G^{[\infty]}_{5,h}(x,y)$ at the central cross section $(x,\bar{y}) = (x,113)$ are compared.



(a) $g_{5,h}(x,y)$ (b) $G_{5,h}^{[\infty]}(x,y)$

Fig. 6. Graphs of the edge detectors $g_{5,h}(x,y)$ and $G_{5,h}^{[\infty]}(x,y)$ with $\beta_5 = 0.7$ at the central cross section $(x,\bar{y}) = (x,113)$, $1 \leq x \leq 223$, of the image in Fig. 4.

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