Syllabus for MATH868C: Several Complex Variables
Fall 2018

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As the name suggests, Several Complex Variables (SCV) is the multi-variable version of complex analysis. However there is important phenomenon that is only seen in the higher dimensional setting, and now represents the building blocks of both complex differential geometry and complex algebraic geometry. The course will consists of 3 units: analytical basics, algebraic topics and geometric topics.

I. We plan to explore how basic information learned in complex analysis generalizes to the higher dimensional setting (maximum principles, Cauchy's formula, Taylor series expansions, etc).

II. Algebraic geometry studies the geometry of the zero set of polynomials. Complex algebraic geometry on the other hand studies the zero sets of holomorphic functions in $\mathbb{C}^n$. In order to investigate these latter objects we introduce the notion of coherent sheaves, and study the local ring of germs of holomorphic functions. We plan to understand the Weierstrass preparation theorems, the coherence theorem of Oka, Remmert's proper mapping theorem and finally Chow's theorem regarding analytic subsets of projective spaces.

III. In the third unit, starting from the Hartogs phenomenon in $\mathbb{C}^n$, we build up to the definition of Stein manifolds. The ultimate aim is to present the L^2 estimates of Hörmander, which have become and indispensable tool in complex differential and algebraic geometry in finding special sections of ample line bundles. Time permitting, we give a characterization of domains of holomorphy in C^n.

Presentation will be self contained, and a good understanding of complex analysis is the only prerequisite of this course. We will mainly use Demailly's "Complex Analytic and Differential Geometry" and Hormander's "An Introduction to Complex Analysis in Several Variables".