MATH868C, Spring 2021
Complex Geometry

Instructor: Tamás Darvas, `tdarvas@umd.edu`.

Class time: TTh 9:30 AM - 10:45 PM.

ZOOM link: `https://umd.zoom.us/j/97851016964`.

Course Website: `www.math.umd.edu/~tdarvas/teaching.html`.

Office hours: after class, or by appointment.

Syllabus: This course is an introduction to complex geometry, with special focus on the geometry of Kähler manifolds. After some introductory material on the calculus of complex manifolds, we propose to cover the following topics, taking time constraints into consideration.

Canonical metrics: there are lots of metrics one can put on a Riemannian manifold. How to choose the best one? In case of compact Kähler manifolds, one can hope to answer this question by imposing conditions on the curvature of the metric, leading to the discussion of Kähler–Einstein metrics, and the Calabi–Yau theorem.

Vector bundles: one can say a lot about sections of positively curved bundles. We will discuss the relationship between Nakano and Griffiths positivity, vanishing theorems, and the Kodaira embedding theorem. Connections with the Griffiths conjecture will be also discussed.

Deformation of complex structures: compared to their smooth counterpart, complex manifolds are quite rigid. When trying to mold one complex structure into another, concrete equations need to be satisfied at the infinitesimal level. Additionally, by a theorem of Kodaira, Kähler manifolds stay stable under deformations. As we will see, the rigidity comes with a blessing: by a theorem of Kodaira-Nirenberg-Spencer, in the absence of cohomological obstructions, infinitesimal deformations are induced by actual deformations. Time permitting, connections with canonical metrics will also be discussed.

Prerequisites: We assume that students are familiar with the theory of smooth manifolds and basic complex analysis. Background in elliptic partial differential equations and basic algebraic geometry will be helpful but not necessary, as we will build many technical tools from the ground up.

Referencees: J.P. Demailly "Complex Analytic and Differential Geometry",
K. Kodaira, J. Morrow "Complex Manifolds",
P. Griffiths, J. Harris "Principles of Algebraic Geometry",
D. Huybrechts "Complex Geometry".

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