AMSC/MATH 674 0101, SPRING 2022

Required text: Evans, *Partial Differential Equations* **second edition**

Recommended: Hörmander, *The analysis of linear partial differential operators*, volume I (any edition). I will provide notes which serve as a guide to the topics in distribution theory we will cover.

Also recommended: Zimmer, *Essential results of functional analysis*, and Wheeden and Zygmund, *Measure and Integral* (real analysis background).

This used to be a "qualifying exam" course on theoretical methods in PDEs, emphasizing methods based on functional analysis and Soboleb space. The formal prerequisite for this course is AMSC/Math 673 (or equivalent), but familiarity with real variables (Math 630) and mathematical maturity are sufficient. Some familiarity with functional analysis would be very helpful, but we will cover all necessary topics.

We will cover the following topics.

$W^{1,p}$ Sobolev spaces in a domain: Definition, extensions, traces, embedding, Poincare's inequality.

H^s Sobolev spaces using the Fourier transform.

Lax-Milgram theorem. Variational solutions to elliptic equations (solving a PDE using the Riesz Representation Theorem in a Hilbert space).

Elements of spectral theory for bounded self-adjoint operators: Definition of the spectrum, the case of compact operator (diagonalization, Fredholm alternative).

There will be several problem sets (some taken from old old qualifying exams, available at http://www-math.umd.edu/quals.html), and 2 in-class exams covering these standard topics.

It is likely we will have enough time left to also cover harmonic analysis/PDE topics such as the Hardy-Littlewood-Sobolev inequality, introduction to Calderon-Zygmund theory for elliptic equations, introduction to Strichartz estimates for the Schrödinger equation, Littlewood-Paley decompositions, examples of global existence and blow-up for the nonlinear Schrödinger equation.

The final exam will be take-home.