Calculus 131, sections 1.1–7.6 Stuff You Need to Know

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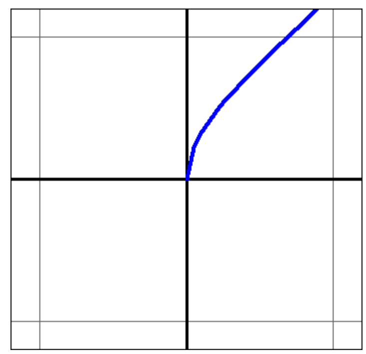
I hope to have notes for each lecture posted on my math department website, http://www.math.umd.edu/~tjp, prior to the lecture itself. Feel free to print out and/or download each of these and bring it with you to class. In this way you can put your attention on listening and thinking, and only need to write all those little "extras" that will come up during my presentation. Need I tell you that these notes will be an outline only, and that they cannot replace your presence in the lecture?

Be sure to attend the discussions on a regular basis, too. You'll find them to be valuable in cementing the topics covered in the lecture. You'll get the most out of the discussion if you do the assigned homework *before* the discussion, and *participate* in all the discussion activities.

To help you get up to speed for Math 131, we're going to spend this first class going over some things I assume you already know, but about which you may need a little reminder. The assigned practice exercises are from the Math 130 Final Exam from Spring 2010. I leave it to you to go back on your own to topics and exercises on which you personally need some more review. Also, you can go to the Math Dept. Testbank (http://db.math.umd.edu/testbank/) and get some other final exams from recent semesters of Math 130.

Example A: Find the following limits: a) $\lim_{x\to 0^+} \sqrt{e^x - 1}$ b) $\lim_{x\to 0^-} \sqrt{e^x - 1}$. (See section 3.1)



From sections 3.4 & 3.5: The following statements are mathematically equivalent:

a) Find the slope of the line tangent to the graph of f at a point (x, y). b) Find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

c) Find the first derivative of $f(x)$. d) Find $f'(x)$. e) Find $\frac{dy}{dx}$ f) Find $D_x[f(x)]$.

Recall, however, that the first derivative is itself a function, which has its own domain and graph. Since it is a function, it has its own derivative. Given a function f, we can calculate the first derivative f' or $\frac{dy}{dx}$. We can then calculate the derivative of f', also called the second derivative of f, symbolically f'' or $\frac{d^2y}{dx^2}$.

<u>Important note</u>: Just like $\frac{dy}{dx}$ is *not* a fraction, but is a notation for the first derivative, $\frac{d^2y}{dx^2}$ is also not a fraction but a notation. *There is no multiplication involved!* Rather, you need to interpret it this way:

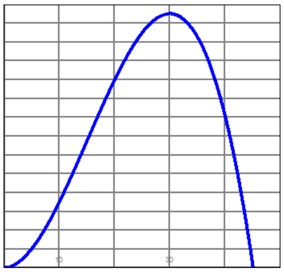
$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$ which means "the derivative of $\frac{dy}{dx}$", the derivative of a derivative.

Example B: Given $f(x) = x^3 - 8x + 2$, find $\frac{d}{dx}[f(x)]$, $D_x^2[f(x)]$, $f(-1)$, $f'(-1)$, and $f''(-1)$.

answers: $3x^2 - 8$, $6x$, 9, –5, –6

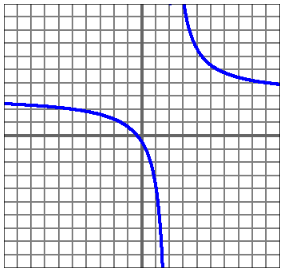
Example C: Public health officials use rates of change to quantify the spread of an epidemic into an equation, which they then use to determine the most effective measures to counter it. A recent measles epidemic followed the equation $y = 45t^2 - t^3$ where y = the number of people infected and t = time in days. a) What is the domain of this function? b) How many people are infected after 5 days? c) What is the rate of spread after 5 days? d) After how many days does the number of cases reach its maximum? e) Use the above to sketch the graph of y.

answers: $0 \le x \le 45$, 1000 people, 375 cases per day, 30 days, see graph pictured to the right,with calculator window set to [0, 50] by [0, 14000]



Example D: Given $y = \left(\sqrt{2x+1}\right)\left(\sqrt{x} - 1\right)$ find $\frac{dy}{dx}$. *answer:* $\frac{4x + 1 - 2\sqrt{x}}{2\sqrt{x(2x+1)}}$

Example E: Given $h(x) = \frac{3x+1}{x-2}$ find h'. *answer:* $\frac{-7}{(x-2)^2}$.

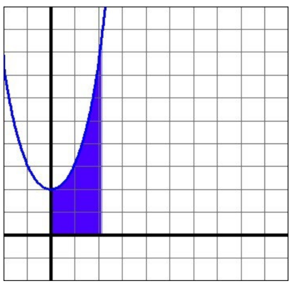


Example F: Given $h(x) = e^{x^2 - x}$, find the first derivative and determine the location of any relative extrema.
answer: $x = \frac{1}{2}$.

Example G: Given $f(x) = \ln\left(x^2 e^x\right)$, find the first and second derivatives. Hint: Use a trig identity.
answers: $\frac{2}{x} + 1$ and $-\frac{2}{x^2}$
Note that domain is not an issue. For *f* and both derivatives, *x* can be any real number except 0.

Example H: Find $\int \left(3x^{-6} - 2e^{5x} + 4x^{-1} - 7\right) dx$. *answer:* $-\frac{3}{5}x^{-5} - \frac{2}{5}e^{5x} + 4\ln|x| - 7x + C$.
Note that domain *is* an issue. For *f* and its integral, *x* can be any real number except 0.

Example I: Find the area under the curve $y = e^x + e^{-x}$ on the interval $0 \le x \le \ln(8)$.
answer: $\frac{63}{8}$ *Note that* $e^{-\ln(8)}$ *can either be evaluated as* $\frac{1}{e^{\ln(8)}} = \frac{1}{8}$ *[negative exponent → reciprocal] or as*
$e^{-\ln(8)} = e^{\ln\left(8^{-1}\right)} = e^{\ln\left(1/8\right)} = \frac{1}{8}$. *[logarithm properties]*



Example J: Determine $\int \tan^2 t\, dt$. Hint: Use a trig identity. *answer*: $\tan t - t + C$.

Example K: Find $\int \frac{2\ln x}{x}\, dx$. *answer*: $(\ln x)^2 + C$.

Example L: $\int \frac{x}{e^{x^2}}\, dx$. *answer*: $-\frac{1}{2}e^{-x^2} + C$.

Example M: $\int \sin^4(x)\cos(x)\, dx$ vs. $\int x^3 \sin\left(x^4\right) dx$. *answers*: $\frac{1}{5}\sin^5(x) + C$, $-\frac{1}{4}\cos\left(x^4\right) + C$