

## Calculus 131, section 8.2 Integration by Parts

notes by Tim Pilachowski

Recall the product rule:  $\frac{d}{dx}[u * v] = u * v' + u' * v$ . From an anti-derivative point of view:

$$\int [u * v' + v * u'] dx = u * v$$

$$\int u dv + \int v du = u * v$$

$$\int u dv = u * v - \int v du$$

giving us integration by parts. We can also perform integration by parts with a definite integral:

$$\int_a^b u dv = [uv]_a^b - \int_a^b v du.$$

The trick is in choosing  $u$  and  $dv$  well: we want the resulting integral to be no more (and hopefully less) intimidating than the original.

Example A: Evaluate  $\int_1^2 x^2 \ln x dx$ . *Answer:*  $\frac{8}{3} \ln 2 - \frac{7}{9}$

Example B:  $\int (x+1)(x-1)^{3/2} dx$ . *Answer:*  $\frac{2}{5}(x+1)(x-1)^{5/2} - \frac{4}{35}(x-1)^{7/2} + C$

Example C:  $\int 2te^{-0.1t} dt$ . *Answer:*  $-20te^{-0.1t} - 200e^{-0.1t} + C$

Example D:  $\int t \sin(6t) dt$ . Answer:  $-\frac{1}{6}t \cos(6t) + \frac{1}{36} \sin(6t) + C$

Example E: Evaluate  $\int x^2 e^x dx$ . Answer:  $e^x(x^2 - 2x + 2) + C$

Example F: Evaluate  $\int 2x e^{x^2} dx$ . Answer:  $e^{x^2} + C$

To summarize the integration we have so far:

If an antiderivative method works, go for it.

If you can identify the product of two functions where one is the derivative of the other, then use substitution.

Otherwise, try integration by parts, in which

$dv$  is the most complicated function that can be integrated—  $\int dv = v$

$u$  can be differentiated

$\int v du$  can be done

$\int u dv = uv - \int v du$ .