Calculus 131, section 10.2-10.3a Addition and Subtraction of Matrices
Scalar Multiplication of a Matrix

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A **matrix** is numbers arranged in rows and columns.

Think "spreadsheet". A name from computer programming that may be familiar to you is "array". Back in the day when I was programming in COBOL we had to arrange our data in "fields". Essentially, a matrix is one method of organizing and arranging data.

Matrices are often/usually named by capital letters italicized. Some examples would be:

The size of a matrix is stated as "number of rows by number of columns". Matrix *A* is a 3 by 1 matrix. *B* is a 3 × 3 matrix. *F* is a 1 by 4 matrix. *M* is a 3 × 4 matrix.

A **square matrix** has the same number of rows and columns. The only square matrix above is *B*.

Matrix *F* is a **row matrix** (size 1 × something); *A* is a **column matrix** (size something × 1).

Two matrices are equal if and only if they are the same size and have matching corresponding row/column elements. For example:

Example A: Solve for the variables *x* and *y*.

$$\begin{bmatrix} -2 & 7 \\ x-4 & -5 \end{bmatrix} = \begin{bmatrix} 2y-1 & 7 \\ 6 & -5 \end{bmatrix}$$

Adding and subtracting matrices is essentially combining like terms: corresponding row/column entries are added together.

Example B. The students in the four 02** discussion sections of the Fall 2011 Math 131 class had the following breakdown of majors and years.

0211 (*M*)	FR	SO	JR	SR
BIO SCI	3	7	1	3
LTSC	0	5	0	1
OTHER	0	3	0	0

0221 (*N*)	FR	SO	JR	SR
BIO SCI	3	11	0	0
LTSC	1	7	0	0
OTHER	0	1	0	0

0231 (*P*)	FR	SO	JR	SR
BIO SCI	2	6	2	0
LTSC	2	10	0	0
OTHER	0	0	1	0

0241 (Q)	FR	SO	JR	SR
BIO SCI	5	3	0	0
LTSC	3	6	1	0
OTHER	0	2	3	0

Rewrite this data into matrices M, N, P and Q where the rows represent majors, columns represent years, and each matrix represents one section.

a) Find $R = M + N + P + Q$ and interpret what it tells us.

b) How many sophomore Biology Science majors are there in the 02** section of the Fall 2011 Math 131 class?

c) How many freshmen are there in the 02** section of the Fall 2011 Math 131 class?

d) How many Letters and Sciences majors are there in the 02** section of the Fall 2011 Math 131 class?

Semi-random notes on matrices:

Any matrices being added *must* be the same size.

Your text introduces the "additive inverse" of a matrix. I'll talk about this as part of subtraction later on.

A "zero matrix" has elements that are all the number 0.

Now we move over to the first topic in section 10.3.

Multiplying a matrix by a scalar (i.e. constant coefficient) is essentially distribution.

Example C:

Given $B = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & 1 & 2 \end{bmatrix}$, find $-2B$.

Example C extended:

Given $B = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 & -2 \\ 0 & -2 & 1 \\ 3 & 0 & -3 \end{bmatrix}$, find $3B - 2C$.

I recommend thinking of subtraction as "adding a negative". Do the scalar multiplication first to make sure that a "minus a negative" isn't missed.