Calculus 131, section 10.3 Multiplication of Matrices

Notes by Tim Pilachowski

Adding and subtracting matrices (section 10.2) is essentially combining like terms. Multiplying a matrix by a scalar (i.e. constant coefficient) is essentially distribution.

Example A:

Given $B = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & 1 & 2 \end{bmatrix}$, find $-2B$. *answer*: $\begin{bmatrix} -6 & -2 & -2 \\ -2 & -2 & 2 \\ -4 & -2 & -4 \end{bmatrix}$

Example A extended:

Given $B = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 & -2 \\ 0 & -2 & 1 \\ 3 & 0 & -3 \end{bmatrix}$, find $3B - 2C$. *do-it-yourself answer*: $\begin{bmatrix} 11 & -1 & 7 \\ 3 & 7 & -5 \\ 0 & 3 & 12 \end{bmatrix}$

Matrix times matrix multiplication is a lot trickier.

Example B-1. I went to a sale at a clothing store and bought 5 pairs of socks, 3 shirts, and 2 pairs of pants. The socks were \$1 each pair, the shirts were \$4 each, and the pairs of pants were \$6 each. How much did I spend?
answer: \$29

Example B-1 revisited: I went to a sale at a clothing store and bought 5 pairs of socks, 3 shirts, and 2 pairs of pants. The socks were \$1 each pair, the shirts were \$4 each, and the pairs of pants were \$6 each. Set up and use matrices to answer the question, "How much did I spend?"

Notes on matrix times matrix multiplication:

Example B-2: I went to a sale at a clothing store and bought 5 pairs of socks, 3 shirts, and 2 pairs of pants. My brother went to the same sale, and got 7 pairs of socks and 8 shirts. The socks were $1 each pair, the shirts were $4 each, and the pairs of pants were $6 each. The regular (non-sale) prices would have been $2, $7 and $10 each respectively. Set up a matrix multiplication and explain what each [row, col] entry in the answer represents.

answer: $NP = \begin{bmatrix} 29 & 51 \\ 39 & 70 \end{bmatrix}$

Notes on this matrix multiplication:

Interpretation of this matrix multiplication:

Theory 1: Is matrix times matrix multiplication commutative, i.e. does $NP = PN$?

Do-it-yourself practice:

$$PN = \begin{bmatrix} 19 & 19 & 2 \\ 69 & 68 & 8 \\ 100 & 98 & 12 \end{bmatrix}$$

Theory 2: In applications, we have to choose our matrix multiplication carefully. How would we know whether to do NP or PN, when both were possible?