Math 131, chapter 12, Probability, Conditional Probability and Bayes' Theorem
supplemental handout prepared by Tim Pilachowski

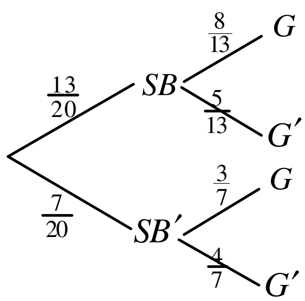
Example 1. Silver Springs, Florida, has a snack bar and a gift shop. The management observes 100 visitors, and counts 65 who eat in the snack bar. Among those who ate in the snack bar, 40 also make a purchase in the gift shop. Of the patrons who did not eat in the snack bar, 15 bought something in the gift shop.

$$P(SB)=\frac{N(SB)}{N}=\frac{65}{100}=\frac{13}{20} \qquad P(SB')=1-\frac{13}{20}=\frac{7}{20}$$

$$P(G \mid SB)=\frac{N(G\cap SB)}{N(SB)}=\frac{40}{65}=\frac{8}{13} \qquad P(G' \mid SB)=1-\frac{8}{13}=\frac{5}{13}$$

$$P(G \mid SB')=\frac{N(G\cap SB')}{N(SB')}=\frac{15}{35}=\frac{3}{7} \qquad P(G' \mid SB')=1-\frac{3}{7}=\frac{4}{7}$$

The tree diagram would look like this.



What is the probability that a person who made a purchase in the gift shop ate in the snack bar?
translation: P(ate in the snack bar | made a purchase) = $P(SB \mid G)$. We need Bayes' Theorem.

Which branches above make up "event G happens" ? the first and third ones down

$$\begin{aligned} P(SB \mid G) &= \frac{P(SB\cap G)}{P(G)} \\ &= \frac{P(SB\cap G)}{P(SB\cap G)+P(SB'\cap G)} \\ &= \frac{P(SB)*P(G\mid SB)}{P(SB)*P(G\mid SB)+P(SB')*P(G\mid SB')} \\ &= \frac{\frac{13}{20}*\frac{8}{13}}{\frac{13}{20}*\frac{8}{13}+\frac{7}{20}*\frac{3}{7}} \\ &= \frac{\frac{2}{5}}{\frac{8}{20}+\frac{3}{20}} \quad =\frac{2}{5}*\frac{20}{11} \quad =\frac{8}{11} \end{aligned}$$

interpretations:
From the given information, $P(G \mid SB)=\frac{8}{13}$ means that 8 out of every 13 people who eat in the snack bar also made a purchase in the gift shop. From Bayes Theorem $P(SB \mid G)=\frac{8}{11}$ means that 8 out of every 11 people who make a purchase in the gift shop also eat in the snack bar.

Example 2: The Gallup organization conducted 10 separate surveys conducted from January through May 2009. At the time of the report, Gallup had found an average of 35% of Americans considering themselves Democratic, 37% independent and 28% Republican. Within those affiliations, the following percentages identified themselves as Conservative, Moderate or Liberal.

	Democrat (event D)	Independent (event I)	Republican (event R)
Conservative (event C)	22%	35%	73%
Moderate (event M)	40%	45%	24%
Liberal (event L)	38%	20%	3%

http://www.gallup.com/poll/120857/conservatives-single-largest-ideological-group.aspx

Results are based on aggregated Gallup Poll surveys of approximately 1,000 national adults, aged 18 and older, interviewed by telephone. Sample sizes for the annual compilations range from approximately 10,000 to approximately 40,000. For these results, one can say with 95% confidence that the maximum margin of sampling error is ±1 percentage point.

This table gives conditional probabilities. $P(\text{Conservative} \mid \text{Independent}) = 35\%$. $P(\text{Liberal} \mid \text{Republican}) = 3\%$.

2-01. Determine $P(D)$, $P(I)$ and $P(R)$.

2-02. Determine $P(D')$, $P(I')$ and $P(R')$.

2-03a. Determine $P(M \mid D)$. b. Write a verbal description of what $P(M \mid D)$ means.

2-04a.Write a verbal description of $D \cap M$. b. Calculate $P(D \cap M)$.

2-05a. Determine $P(M' \mid R)$. b. Write a verbal description of what $P(M' \mid R)$ means.

2-06a. Write a verbal description of $R \cap M'$. b. Calculate $P(R \cap M')$.

2-07. Draw a tree diagram to illustrate the events and probabilities for this two-stage experiment.

2-08. Calculate $P(M)$.

2-09a. Write a verbal description of $D \cup M$. b. Calculate $P(D \cup M)$.

2-10a. Use Bayes' Theorem to calculate $P(D \mid M)$. b. Write a verbal description of what $P(D \mid M)$ means.

2-11. Are D and M independent events? How do you know?

2-12. Calculate $P(M')$.

2-13a. Write a verbal description of $R \cup M'$. b. Calculate $P(R \cup M')$.

2-14a. Use Bayes' Theorem to calculate $P(R \mid M')$. b. Write a verbal description of what $P(R \mid M')$ means.

2-15. Are R and M' independent events? How do you know?

Example 3: According to U.S. Department of Labor employment reports, in 2010, 39.3% of jobs were classified as Management, professional, and related occupations (Mgt), 14.5% were Service occupations (Svc), 23.2% were Sales and office occupations (SO), 9.9% were Natural resources, construction, and maintenance occupations (NCM), and 13.1% were Production, transportation, and material moving occupations (PTM). Among Mgt, 48.6% were held by male workers, and 51.4% were held by female workers. The other categories were: Svc, 50.6% male, 49.4% female; SO, 38.3% male, 61.7% female; NCM, 95.9% male, 4.1% female; PTM, 80.2% male, 19.8% female.

source: United States Department of Labor, Bureau of Labor Statistics report "Median weekly earnings of full-time wage and salary workers by detailed occupation and sex" covering the year 2010. ftp://ftp.bls.gov/pub/special.requests/lf/aat39.txt

Let A = Management/professional, B = Service, C = Sales/office, D = Natural resources/construction/ maintenance, E = Production/transportation, M = male and F = female.

3-01a. Determine the probability that a person was not in a Management/Professional position. b. Determine the probability that a person was not in a Sales/Office position.

3-02. Determine the probability that a person was male given that his job was classified as a service occupation.

3-03. Calculate the probability that an individual was a man working in a service occupation.

3-04. Determine the proportion of natural resources/construction/maintenance workers that were female.

3-05. Determine the probability that a person was a woman employed as a natural resources/construction/ maintenance worker.

3-06. Draw a tree diagram to illustrate the events and probabilities given above.

3-07a. Calculate the proportion of the 2010 U.S. workforce that was male. b. Calculate the proportion of the 2010 U.S. workforce that was female.

3-08. Calculate the probability that an individual is either a man or working in a service occupation.

3-09. Determine the probability that a person was either female or was employed as a natural resources/construction/ maintenance worker.

3-10a. Calculate $P(D \mid F)$. b. Write a verbal description of what $P(D \mid F)$ means. c. Are D and F independent events? How do you know?

3-11a. Among the male members of the U.S. workforce, what proportion were in service occupations? b. Are being male and working in a service occupation independent events? How do you know?

Selected Answers to Example 2

2-01. 0.35, 0.37, 0.28; 2-02. 0.65, 0.63, 0.72; 2-03. 0.4, probability that a person is Moderate given that he/she is a Democrat; 2-04. A person is both a Democrat and Moderate., 0.14; 2-05. not a Moderate = either Conservative or Liberal, 0.76; 2-06. A person is a Republican and not Moderate, *or* A person is a Republican and either a Conservative or a Liberal., 0.2128; 2-08. 0.3737; 2-09. either a Democrat or a Moderate or both., 0.5837; 2-10. ≈ 0.3746, probability that a person is a democrat given that he/she is a Moderate; 2-11. not independent since $P(D \mid M) \neq P(D)$; 2-12. 0.6263; 2-13. A person is either a Republican or not a Moderate. *or* A person is either a Republican or a Conservative or a Liberal., 0.6935; 2-14 ≈ 0.3398, probability that a person who is not a Moderate is also a Republican; 2-15. not independent since $P(R \mid M^c) \neq P(R)$.

Selected Answers to Example 3

3-01. 0.607, 0.768; 3-02. 0.145; 3-03. ≈ 0.073; 3-04. 0.041; 3-05. ≈ 0.004; 3-07. ≈ 0.553, 0.447; 3-08. ≈ 0.625; 3-09. ≈ 0.542; 3-10. ≈ 0.009, About 0.9% (almost 1%) of employed women work in Natural Resources, Construction or Maintenance occupations., not independent since $P(D \mid F) \neq P(D)$. Note that if we had rounded off to 2 decimal places we would have reached an incorrect conclusion.; 3-11. ≈ 0.132, About 13% of employed men work in Service occupations., They are not independent since $P(B \mid M) \neq P(B)$.