Calculus 131, section 13.2 Continuous Random Variables, E(x) & Var(x)

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Recall from section 12.4, for *discrete* random variables:

$$\mathrm{E}(X) = \mu = x_1\,P(x_1) + x_2\,P(x_2) + x_3\,P(x_3) + \ldots + x_n\,P(x_n) = \sum_x x\,P(x) = \text{ sum of [value * probability]}$$

$$\mathrm{Var}(X) = (x_1-\mu)^2\,P(x_1) + (x_2-\mu)^2\,P(x_2) + (x_3-\mu)^2\,P(x_3) + \ldots + (x_n-\mu)^2\,P(x_n) = \sum_x (x-\mu)^2\,P(x)$$

$$= \text{sum of [(value – mean)}^2 \text{ * probability].}$$

If we apply the same underlying concept to continuous random variables, we get analogous integrals. Given a random variable X with a probability density function $f(x)$ on an interval $a \le x \le b$:

$$\mathrm{E}(x) = \mu = \int_a^b x\,f(x)\,dx \qquad\qquad \mathrm{Var}(x) = \int_a^b [x-\mu]^2\,f(x)\,dx = \int_a^b x^2\,f(x)\,dx - \mu^2 .$$

The text verifies the second, easier, computational formula for Var(x), so I won't repeat that proof here.
As before, standard deviation of $x = \sigma = \sqrt{\mathrm{Var}(x)}$.

Example A: Find the expected value and variance for the uniform probability density function $f(x) = \frac{1}{10},\ 0 \le x \le 10$. *answers*: 5, $\frac{25}{3}$

Example B: Find the expected value and variance for the probability density function $f(x) = 2x - 2,\ 1 \le x \le 2$.
I'll leave it to you to verify that f is a probability density function. *answers*: $\frac{5}{3}$, $\frac{1}{18}$

Example C: The monthly demand for a product (continuous random variable *X*) has a probability density function $f(x)=\frac{1}{36}(6x-x^2)$, $0 \le x \le 6$. Find expected monthly demand and the probability that the random variable is within 1 standard deviation of the mean. *answers*: 3, ≈ 0.6261

Example D: For a particular machine, its useful lifetime is modeled by $f(t)=0.1e^{-0.1t}$, $0 \le t \le \infty$. Find the expected number of years that the machine will last, along with the standard deviation. *answers*: 10, 10