Math 131 Exam 1 Sample 2 Solution OUTLINE

1. (a) Draw a number line with marks at $x = 1, 1.5, 2, 2.5, 3, 3.5, 4$ and apply the rule.

   (b) i. At R2 a cook makes \$15/hour.

   ii. During W3 each restaurant hires 220 hours' worth of waitresses.

   iii. If A is the first matrix and B is the second we'd do BA. This is because B has (restaurants $\times$ employee type) and A has (employee type $\times$ week). The result BA is then (restaurants $\times$ week). The units are dollars/week. Consequently BA gives dollars/week for each restaurant for each week in a three-week period.

2. (a) Rewrite as $\int 4xe^{-x}\,dx$ and use IBP with $u = 4x$ and $dv = e^{-x}\,dx$.

   (b) Evaluate $\frac{1}{10-0}\int_0^{10} Pe^{0.05x}\,dx$. The result will have a P in it. Set it equal to 5000 and solve for P to get $P = 2500/(e^{0.5} - 1)$.

3. (a) First do $\int \frac{2x}{(x^2+1)^{3/2}}\,dx$ by substituting $u = x^2 + 1$, getting $-\frac{2}{\sqrt{x^2+1}} + C$. Rewrite $\int_2^{\infty} \frac{2x}{(x^2+1)^{3/2}}\,dx$ with a limit, put in the integral and then the b and take the limit as $b \to \infty$.

   (b) This is awkwardly written. The matrices A and B themselves do not need to be square, for example if A is 3×5 and B is 5×3 then both AB and BA are permissible. The resulting AB and BA will be square though.

4. (a) Write is as a matrix and reduce:

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 7 \end{bmatrix} \to \dots \to \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

   So $x = -2$ and $y = -1$.

   (b) If $A = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$ then the solution is $\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}\begin{bmatrix} 1 \\ 7 \end{bmatrix}$. Use the shortcut to get A^{-1}.

5. (a) We augment and reduce. It's pretty quick:

$$\begin{bmatrix} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{bmatrix} \to \dots \to \begin{bmatrix} 1 & 0 & 0 & 1 & 2 & -2.5 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0.5 \end{bmatrix}$$

   So the inverse the 3×3 part on the right.

   (b) We solve $(0.3 - \lambda)(0.3 - \lambda) - (0.5)(0.5) = 0$ to get $\lambda = -0.2, 0.8$. We use the positive one. We reduce

$$\begin{bmatrix} -0.5 & 0.5 \\ 0.5 & -0.5 \end{bmatrix} \to \dots \to \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

   so that y is anything and $x = y$. To total 4000 then we need $x = 2000$ and $y = 2000$. The population is shrinking each year to 0.8 of its original size.