

Directions: Do not simplify unless indicated. Non-graphing calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem. All real-world problems should include units.

**Please put problem 1 on answer sheet 1**

1. (a) Given  $f(x, y, z) = x^2y - x^3z + x \ln y$  find  $f(-2, 1, 3)$ . [5]

(b)  $\int_0^6 \int_0^{x^2} (x+y) dy dx$  [15]

**Please put problem 2 on answer sheet 2**

2 (a) The surface area of a human body (in  $m^2$ ) is approximately  $A = 0.202W^{0.425}H^{0.725}$ , where  $W$  is the weight of the person in kilograms and  $H$  is the height in meters. Find an equation that represents how a person's surface area will change as her or his weight changes, assuming that height remains constant. [8]

(b) For  $g(x) = xy - e^{xy}$  find  $g_x$  and  $g_{xy}$ . [12]

**Please put problem 3 on answer sheet 3**

3. (a) Let  $f(x, y) = x^4 - 10xy + \frac{1}{2}y^2 - 7$ . It can be shown that the first partial derivatives of  $f(x, y)$  are zero at  $(5, 50)$ . Use the second derivative test to determine the nature of the function at this point. (Write the appropriate term: *relative maximum*, *relative minimum*, *neither a max nor a min (saddle point)*, or *inconclusive*.) [10]

(b) Solve the differential equation  $(t^2 + 1)\frac{dy}{dt} = t(y + 1)$ . [10]

**Please put problem 4 on answer sheet 4**

4. (a) The temperature of a frozen pizza rises at a rate expressed by the differential equation  $y' = 9(400 - y)$ . A pizza with a temperature of  $30^\circ$  F is put into the oven. Solve to find an equation which expresses the pizza's temperature  $y$  as a function of time  $t$  in minutes. [10]

(b) A population of wolverines is modeled by the differential equation  $\frac{dy}{dt} = 0.02y + 16t$ . Solve to find an equation to represent population  $y$  at time  $t$  (in years) given an initial population of 10,000 wolverines. [10]

**Please put problem 5 on answer sheet 5**

5. (a) Let  $f(t)$  be the solution to  $y' = y + 4t^2$ ,  $y(1) = 6$ . Use Euler's method with  $n = 2$  to estimate  $f(2)$ . [8]

(b) Given the system of linear differential equations,  $\frac{dx_1}{dt} = 3x_1 - 2x_2$  and  $\frac{dx_2}{dt} = x_1 + e^t$ , a student has found the necessary eigenvalues and eigenvectors to be  $\lambda_1 = 1 \Rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $\lambda_2 = 2 \Rightarrow \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ . Do the next steps of the solution process to answer  $\frac{dy_1}{dt} = ?$  and  $\frac{dy_2}{dt} = ?$  Do not solve these two DEs! [12]

**The End**