Math 131 Exam 2 Sample 2 **Fall 2010**

Directions: Do not simplify unless indicated. Non-graphing calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem. All real-world problems should include units.

Please put problem 1 on answer sheet 1

1. (a) The number of cows that can graze on a ranch is approximated by $f(x, y) = 9x + 5y - 5$, where x is the number of acres of grass and y the number of acres of alfalfa. First find $f(40, 75)$, then write a sentence explaining what your result means. [8]

(b) Find and categorize all relative maximum or minimum points of $g(x, y) = -x^2 - y^2 + 6x + 8y - 21$. [12]

Please put problem 2 on answer sheet 2

2. (a) Given $h(x, y, z) = x^2 y - x^3 z + x \ln y$ find $h_y(-2, 1, 3)$. [8]

(b) For the unknown function $m(x, y)$, the *first partial derivative* $m_y = x \ln(x^2 - xy)$. Find m_{yx}. [12]

Please put problem 3 on answer sheet 3

3. (a) Find the volume under the surface $z = 6x^2 y$ and above the rectangle $0 \le x \le 4,\ 0 \le y \le 3$. [15]

(b) The rate of change of the population of Binthar, Montana, is given by $\frac{dy}{dt} = 0.02y$, where y is the population in thousands at time t, in years. Let $t = 0$ represent the year 2000, when the population was 300,000. Find the population function. Also state whether the population is growing or declining. (*Hint: If you recognize this one, you can write the function without solving the DE.)* [5]

Please put problem 4 on answer sheet 4

4. (a) Use the first-order linear process to solve the differential equation $\frac{dy}{dx} + y\cos x = \cos x$ with the initial value condition $y(0) = 3$. [10]

(b) A researcher finds that the rate of productivity of worker bees in a newly-established hive is related to the size of the colony by the differential equation $\frac{dy}{dx} = y(x^2 + 1)$ with initial condition $y(0) = 2$. Solve to find an equation which represents the amount the worker bees produce as a function of colony size x. [10]

Please put problem 5 on answer sheet 5

5. (a) Let $f(t)$ be the solution to $y' = y^2 + 4t, \quad y(1) = 2$. Use Euler's method with $n = 2$ to estimate $f(2)$. [8]

(b) Given the system of linear differential equations, $\frac{dx_1}{dt} = 3x_1 - 2x_2$ and $\frac{dx_2}{dt} = x_1 + e^t$, a student has found the necessary eigenvalues and eigenvectors to be $\lambda_1 = 1 \Rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\lambda_1 = 2 \Rightarrow \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and has derived $\frac{dy_1}{dt} - y_1 = 2e^t$ and $\frac{dy_2}{dt} - 2y_2 = -e^t$. Finish the solution process to answer $x_1(t) = ?$ and $x_2(t) = ?$ [12]

The End