**Math 131 Exam 3 Sample 1** **Fall 2010**

**Directions:** Do not simplify unless indicated. Non-graphing calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem. All real-world problems should include units.

**Please put problem 1 on answer sheet 1**

1. (a) Suppose $P(E) = 0.3$, $P(F) = 0.4$ and $P(E \cap F) = 0.1$.
   i. Find $P(E')$
   ii. Find $P(E \cup F)$
   iii. Are $E$ and $F$ independent? Justify.
   iv. Find $P(E \cup F')$

   (b) Suppose the event $A$ corresponds to a person having a disease and the event $B$ corresponds to the test for that disease giving a positive result. Suppose Joe went to the doctor and tested positive. Would Joe be most interested in $P(A|B)$ or $P(B|A)$? Explain.

**Please put problem 2 on answer sheet 2**

2. (a) A bowl contains 3 red marbles, 7 blue marbles and 2 purple marbles. You remove two without replacement. Let $A$ be the event corresponding to the first being blue and let $B$ be the event corresponding to the second being purple. Find $P(A \cap B')$ and explain what this probability means in real-world terms.

   (b) Suppose the TSA is doing a study. For any given passenger let $E$ be the event corresponding to that passenger looking nervous and $F$ be the event corresponding to that passenger carrying a weapon. The TSA finds out that:
   - There is a 1% chance that a passenger carries a weapon.
   - If a passenger is carrying a weapon there is a 90% chance he will look nervous.
   - If a passenger is not carrying a weapon there is a 10% chance he will look nervous.

   Use Bayes' Theorem to find out the probability that a nervous-looking passenger actually has a weapon. In light of this is it a good idea to do a second screening for nervous passengers? Justify.

**Please put problem 3 on answer sheet 3**

3. The time between two calls at a call center is exponentially distributed with mean 2 minutes.
   (a) Write down the probability density function for this distribution.
   (b) What's the probability that two calls will come in under 1 minute apart? Approximate to one decimal digit.
   (c) What's the probability that two calls will come in over 5 minutes apart? Approximate to one decimal digit.

**Please put problem 4 on answer sheet 4**

4. Suppose the government is trying to decide whether to legalize a certain life-extending drug. Laboratory results indicate that a single dose of the drug increses life by a random variable with mean 5 years and standard deviation 3 years.
   (a) Suppose the drug is administered to 100 patients. Use the Central Limit Theorem to approximate the probability of the average lifespan being increased by at least 5.6 years.
   (b) Suppose the drug is administered to 400 patients. Up to how many years will the lifespan of 90% of people increase?

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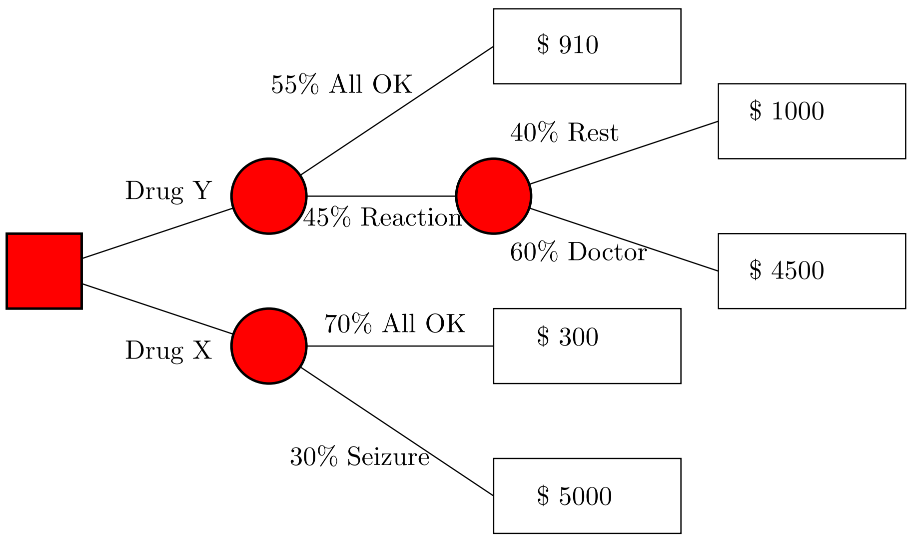
**Please put problem 5 on answer sheet 5**

5. (a) A continuous random variable $X$ has probability density function given by $f(x) = \frac{1}{2\sqrt{x}}$ on $[1, 4]$.
   Find $E(X)$, $Var(X)$ and $\sigma(X)$.

   (b) Use the folding-back (pruning) method to make the cheapest decision on a health-care choice for individuals with a sinus infection using the following tree. The dollar figure indicates cost. Is drug X or drug Y the cheapest choice? Explain how you might justify this to a critic who points out the negative aspect of your choice (both choices have a negative aspect).




**Formulas You Might Find Helpful**

- $P(E|F) = \frac{P(E\cap F)}{P(F)}$
- $P(F|E) = \frac{P(F)P(E|F)}{P(F)P(E|F)+P(F')P(E|F')}$.
- $\Sigma x P(x)$
- $\Sigma(x-\mu)^2 P(x)$
- $\sqrt{Var(X)}$
- $\int_a^b x f(x)\ dx$
- $\int_a^b (x-\mu)^2 f(x)\ dx = \int_a^b x^2 f(x)\ dx - \mu^2$
- $f(x) = ae^{-ax}$