Math 131 Exam 3 Sample 2 **Fall 2010**

Directions: Do not simplify unless indicated. Non-graphing calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem. All real-world problems should include units.

Please put problem 1 on answer sheet 1

1. (a) Suppose $P(E) = 0.5$, $P(F) = 0.6$ and $P(E \cap F) = 0.4$.
   i. Draw a Venn diagram and fill in all four probabilities.
   ii. Find $P(E|F)$
   iii. Are E and F mutually exclusive? Justify.
   iv. Redraw your picture without probabilities and shade in $E' \cup F$.

   (b) Suppose the event A corresponds to a person having a disease and the event B corresponds to the test for that disease giving a positive result. Suppose Joe has that disease. Would the test manufacturer be most interested in $P(A|B)$ or $P(B|A)$? Explain.

Please put problem 2 on answer sheet 2

2. (a) You are taking part in a study in which you have 100 patients. You know that 20 have condition A, 50 have condition B and 30 have condition C. You don't know which patients have which conditions. You pick a first patient and then a second. What is the probability that the first has condition A and the second does not have condition C?

   (b) Suppose you are collecting data on a group of people who may or may not have a certain allele (a form of a gene). Let E be the event that a person has that allele and let F be the event that your test indicates that a person has that allele. You are interested in the probability that a person who tests positive for the allele actually has it. Unfortunately your graduate student is sloppy and only finds out:
   - There is a 0.002 probability that a person has the allele.
   - If a person has the allele then there is a 0.93 probability that the test will be positive.
   - If a person does not have the allele then there is a 0.05 probability that the test will be positive.

   Use Bayes' Theorem to find out the probability that person who tests positive actually has the allele.

Please put problem 3 on answer sheet 3

3. Among 70 year-olds the *additional* lifespan is exponentially distributed with mean 5 years.
   (a) What is the probability density function for this distribution?
   (b) What's the probability that a randomly chosen 70-year old will live to at least 75? Approximate to one decimal place.
   (c) Under how many additional years will 90% of 70-year old live? Approximate to one decimal place.

Please put problem 4 on answer sheet 4

4. You are doing a laboratory experiment with a certain organism. The lifespan of this organism has mean 2 months and standard deviation 1 month.
   (a) You collect a sample of 50 organisms. Use the CLT to approximate the probability that the average lifespan of all your organisms is between 1.7 and 2.1 months.
   (b) If your sample size is 100 organisms find the age above which only 10% will live.

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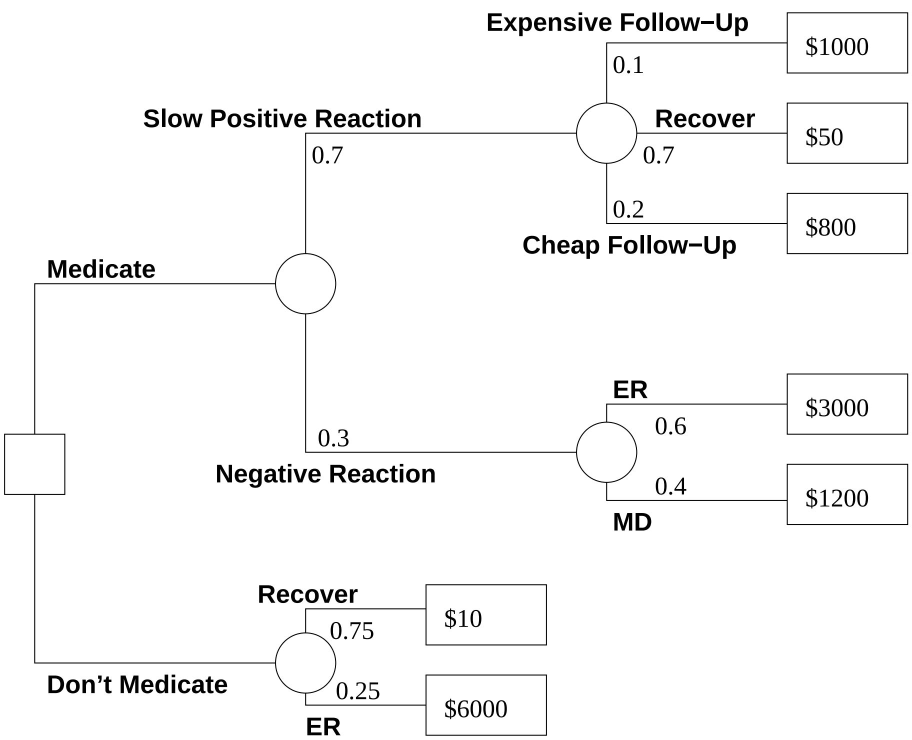
Please put problem 5 on answer sheet 5

5. (a) Define $f(x) = kx^2$ on $[0, 5]$.
   i. Find k so that $f(x)$ is a probability density function.
   ii. Find $E(X)$.
   iii. Find $Var(X)$.

   (b) Use the folding-back (pruning) method to make the cheapest decision on whether to medicate for a certain condition or not.



Formulas You Might Find Helpful

- $P(E|F) = \frac{P(E \cap F)}{P(F)}$
- $P(F|E) = \frac{P(F)P(E|F)}{P(F)P(E|F)+P(F')P(E|F')}$.
- $\Sigma x P(x)$
- $\Sigma (x-\mu)^2 P(x)$
- $\sqrt{Var(X)}$
- $\int_a^b x f(x)\ dx$
- $\int_a^b (x-\mu)^2 f(x)\ dx = \int_a^b x^2 f(x)\ dx - \mu^2$
- $f(x) = ae^{-ax}$