Math 131 Final Exam Sample 1 **Fall 2010**

Directions: Do not simplify unless indicated. Non-graphing calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem. All real-world problems should include units.

Please put problem 1 on answer sheet 1

1. (a) Suppose nutritional supplement I contains 5 mg of niacin and 2 mg of fish oil while nutritional supplement II contains 2 mg of niacin and 6 mg of fish oil. You take x units of I and y units of II. Write down the system of equations which would correspond to needing A mg of niacin and B mg of fish oil, where A and B are unknown constants. Solve using the inverse of a matrix.

   (b) Suppose the Leslie Matrix for a population of juveniles and adults is shown below.

$$\begin{bmatrix} 0.5 & -0.1 \\ 0.2 & 0.2 \end{bmatrix}$$

   If there are 500 adults in a stable population then how many juveniles are there?

Please put problem 2 on answer sheet 2

2. (a) The calorie requirement for a certain athlete is $C(x, y) = 4xy^2 + e^{0.01xy}$ where x is the athlete's age in years and y is the number of hours she exercises daily.

      i. Find, simplify and interpret $C(16, 6)$.

      ii. Find, simplify and interpret $C_x(20, 4)$.

      iii. Suppose you set $C_y(x, 18) = 500$ and solved for x. Without actually doing this (don't!), explain what you would be finding.

   (b) Find and categorize all relative maxima, minima and saddle points for $f(x, y) = 3x^2 - 6xy + y^3 - 9y$.

Please put problem 3 on answer sheet 3

3. (a) Show that the eigen-pairs for $M = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$ are $\lambda = 2$ with $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\lambda = 1$ with $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

   Note: Yes, I am giving you the answers - you are expected to show the work!

   (b) Solve the following system of differential equations.

$$\frac{dx_1}{dt} = 4x_1 - 3x_2$$
$$\frac{dx_2}{dt} = 2x_1 - x_2 + 1$$

Please put problem 4 on answer sheet 4

4. (a) Suppose the rate of growth on day t of a certain population is given by $r(t) = t \ln t$. Use the Trapezoidal Rule with $n = 4$ to approximate the change in population between $t = 1$ and $t = 3$.

   (b) Suppose an operation has a 40% chance of costing \$1000 and a 60% chance of costing \$2000.

      i. Suppose X is the random variable for the cost of one operation. Find $E(X)$ and $\sigma(X)$.

      ii. Suppose 100 operations take place. Use the CLT to approximate the probability that the average cost will be \$1680 or more.

      iii. Suppose 100 operations take place. Use the CLT to approximate the maximum cost of the cheapest 25% of the operations.

Turn Over

Please put problem 5 on answer sheet 5

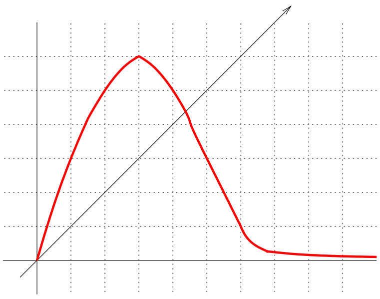
5. (a) Suppose the lifespan of a chimpanzee is exponentially distributed with average 20 years.
   i. What is the probablity that a chimpanzee will live over 15 years?
   ii. Over how many years will just 5% of chimpanzees live?

   (b) A bag contains 5 red balls and 10 yellow balls. You remove two without replacement.
   i. What is the probability that the first is red and the second is red?
   ii. What is the probability that the first is not red and the second is red?
   iii. What is the probability that the second is red?

Please put problem 6 on answer sheet 6

6. (a) Suppose a discrete population has iterative function $f(x) = 2.5x(1 - x)$.
   i. Find the fixed point.
   ii. Determine using the derivative whether the fixed point is stable or not.
   iii. Suppose $a_1 = 0.5$. Find a_2 through a_5. Approximate each to two decimal digits.

   (b) Suppose the iterative function for a discrete population is shown here. The grid is at 1 units.



   i. What is the approximate fixed point?
   ii. Assuming that $a_1 = 2$, cobweb until you get into a never-ending cycle which repeats three values. All should be exact! Write down your values (up to those three) and also recopy your cobweb diagram as well as you can onto the answer sheet.
   iii. If a_n represents the number of rabbits in thousands during year n explain in words what is happening to the rabbit population in the long term.
   iv. Suppose a_1 is very large. Explain what you can say about the next few iterations.

The End