Math 131 Final Exam Sample 2 **Fall 2010**

Directions: Do not simplify unless indicated. Non-graphing calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem. All real-world problems should include units.

Please put problem 1 on answer sheet 1

1. (a) Suppose we are analyzing two vitamins (A and B) in two nutritional supplements (I and II) taken by an individual (Bill). Consider the following two tables. The first contains how many of each supplement Bill took while the second contains the amount of each vitamin per supplement

# supp	Bill
I	3
II	5

mg/supp	I	II
A	20	15
B	10	30

   i. What does the 20 represent?
   ii. What does the 5 represent?
   iii. Put these tables into matrices and multiply them in such a way that the result is meaningful. Describe what the resulting matrix represents.

   (b) Suppose the Leslie Matrix for a population of juveniles and adults is shown below:

$$\begin{bmatrix} 0.2 & 0.7 \\ 0.3 & 0.6 \end{bmatrix}$$

   i. If there are 1000 juveniles and 600 adults (the vector $\begin{bmatrix} 1000 \\ 600 \end{bmatrix}$) at first, how many will there be after one iteration and after two iterations?
   ii. Find a population of 2000 for which the ratio of juveniles to adults stays constant.

Please put problem 2 on answer sheet 2

2. (a) Evaluate $\int_0^2 \int_0^{x+1} x + y \; dy \; dx$. Simplify.

   (b) Find and categorize all relative maxima, minima and saddle points for

$$f(x, y) = x^3 - 6x^2 + 6y - y^2$$

Please put problem 3 on answer sheet 3

3. (a) A population y grows by two methods. Reproduction has a 3.2% rate but in addition 200 new individuals move in each year. If the initial population was 1000, write down the initial value problem corresponding to this scenario. Do not solve.

   (b) Solve implicitly: $\frac{dy}{dx} = \frac{2\sqrt{y}}{x} + \sqrt{y}$

   (c) Solve implicitly: $t\frac{dy}{dt} = t^3 - t - y$

Please put problem 4 on answer sheet 4

4. Suppose you have an experiment which involves flipping three coins and counting the heads. Let X be the number of heads.

   (a) Write down a probability distribution table for this.
   (b) Find $P(X \leq 2)$.
   (c) Show that $E(X) = 1.5$ and $\sigma(X) = 0.75$.
   (d) Suppose you repeat this experiment 100 times and average all the results. What is the probability that your average is between 1.4 and 1.65? Use the CLT.
   (e) Above what value is there only a 5% probability that your average will lie? Use the CLT.

Turn Over

Please put problem 5 on answer sheet 5

5. (a) The time between successive cell divisions in an organism is exponentially distributed. You monitor this organism and notice that 20% of the time there is a 6 or more minute wait between divisions, find the mean division time. Approximate to two decimal digits.

   (b) A bag contains 2 red balls and 4 yellow balls. You remove two without replacement. Let E be the even that the first is red and let F be the even that the second is red. Use Bayes' Theorem to find the probability that the first is red given that the second is red.

   (c) Draw a Venn diagram (with probabilities) for two events which are independent but not mutually exclusive.

Please put problem 6 on answer sheet 6

6. (a) Suppose a discrete population has iterative function $f(x) = \frac{1}{16}x(x-1)^2$.

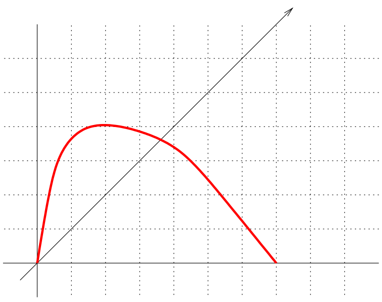
   i. Find the fixed point.

   ii. Suppose $a_1 = 5.1$. Find a_2 through a_5. Approximate each to one decimal digit.

   iii. Based upon your answer to ii determine if the fixed point is stable or not.

   iv. Determine using the derivative whether the fixed point is stable or not.

   (b) Suppose the iterative function for a discrete population is shown here.



   i. What is the approximate fixed point?

   ii. Assuming that $a_1 = 0.5$, cobweb until you get reasonably close to (pretty much on top of) the fixed point. Write down your values (reasonable approximations suffice) and also recopy your cobweb diagram as well as you can onto the answer sheet.

   iii. Find a_1 so that a_2 is the fixed point.

   iv. What is the largest that a_1 could be? Why?

   v. What is the largest that a_2 could be? Why?

The End