Solutions to MATH141 Quiz 1

September 21, 2009

12 PM

Problem 1

Use the disk method

$$V = \pi \int_0^{\pi/6} \left(\sqrt{\cos(x)}\right)^2 dx = \pi \int_0^{\pi/6} \cos(x)dx = \pi \sin(x)\Big|_0^{\pi/6} = \frac{\pi}{2}.$$

Problem 2

Compute the derivative: $f'(x) = 2$.
Use the formula for length

$$L = \int_1^5 \sqrt{1+2^2}dx = \int_1^5 \sqrt{5}dx = \sqrt{5}x\Big|_1^5 = 4\sqrt{5}.$$

Problem 3

Verify $f \geq g$ on the interval $[2,5]$
$f(x) - g(x) = (2x-1) - (x-2) = 2x - 1 - x + 2 = x + 1 \geq 0$ on $[2,5]$.

$$\begin{aligned} M_y &= \int_2^5 x(x+1)dx = \int_2^5 (x^2+x)dx = \frac{x^3}{3}\Big|_2^5 + \frac{x^2}{2}\Big|_2^5 \\ &= \left(\frac{125}{3} - \frac{8}{3}\right) + \left(\frac{25}{2} - \frac{4}{2}\right) = \left(\frac{117}{3} + \frac{21}{2}\right) = \frac{297}{6} = \frac{99}{2}. \end{aligned}$$

Problem 4

Plot a few points:

t	0	$\pi/2$
x	0	-1
y	1	$\pi/2$

The direction of the parametrization is counterclockwise.

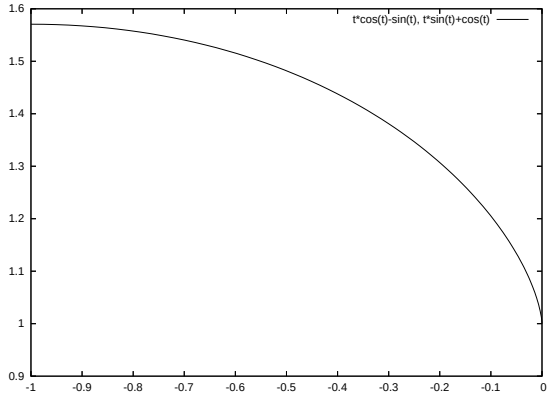

Figure 1: Graph for Problem 4, 12pm.

1 PM

Problem 1

Use the disk method

$$V = \pi \int_0^{\pi/2} \left(\sqrt{\cos(x)}\right)^2 dx = \pi \int_0^{\pi/2} \cos(x) dx = \pi \sin(x)\Big|_0^{\pi/2} = \pi.$$

Problem 2

Compute the derivative: $f'(x) = 2$.
Use the formula for length

$$L = \int_1^5 \sqrt{1 + 2^2} dx = \int_1^5 \sqrt{5} dx = \sqrt{5}x\Big|_1^5 = 4\sqrt{5}.$$

Problem 3

Verify $f \geq g$ on the interval $[2, 5]$
$f(x) - g(x) = (2x + 1) - (x + 2) = 2x + 1 - x - 2 = x - 1 \geq 0$ on $[2, 5]$.

$$\begin{aligned} M_y &= \int_2^5 x(x-1)dx = \int_2^5 (x^2 - x)dx = \frac{x^3}{3}\Big|_2^5 - \frac{x^2}{2}\Big|_2^5 \\ &= \left(\frac{125}{3} - \frac{8}{3}\right) - \left(\frac{25}{2} - \frac{4}{2}\right) = \left(\frac{117}{3} - \frac{21}{2}\right) = \frac{171}{6} = \frac{57}{2}. \end{aligned}$$

Problem 4

Plot a few points:

t	0	$\pi/2$	π
x	0	1	$\pi/2$
y	1	$\pi/2$	-1

The direction of the parametrization is clockwise.

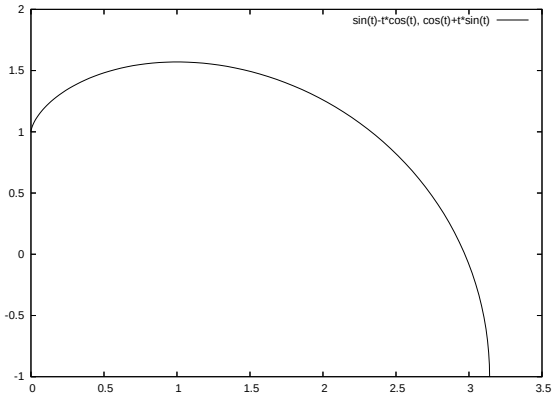

Figure 2: Graph for Problem 4, 1pm.

2 PM

Problem 1

Use the disk method

$$V = \pi \int_0^2 \left(\sqrt{x+1}\right)^2 dx = \pi \int_0^2 (x+1)dx = \pi \frac{(x+1)^2}{2}\Big|_0^2 = \pi\left(\frac{9-1}{2}\right) = 4\pi.$$

Problem 2

Compute the derivative: $f'(x) = 1$.
Use the formula for length

$$L = \int_1^4 \sqrt{1+1^2}dx = \int_1^4 \sqrt{2}dx = \sqrt{2}x\Big|_1^4 = 3\sqrt{2}.$$

Problem 3

Verify $f \geq g$ on the interval $[2,5]$
$f(x) - g(x) = (2x+1) - (x+2) = 2x + 1 - x - 2 = x - 1 \geq 0$ on $[2,5]$.

$$\begin{aligned} M_x &= \frac{1}{2}\int_2^5 \left[(2x+1)^2 - (x+2)^2\right] dx = \frac{1}{2}\int_2^5 (4x^2 + 4x + 1 - x^2 - 4x - 4)dx \\ &= \frac{1}{2}\int_2^5 (3x^2 - 3)dx = \frac{1}{2}x^3\Big|_2^5 - \frac{3x}{2}\Big|_2^5 \\ &= \frac{125-8}{2} - \frac{15-6}{2} = \frac{117-9}{2} = \frac{108}{2} = 54. \end{aligned}$$

Problem 4

Plot a few points:

t	0	$\pi/2$	π
x	0	$\pi/2-1$	π
y	0	1	2

The direction of the parametrization is clockwise.

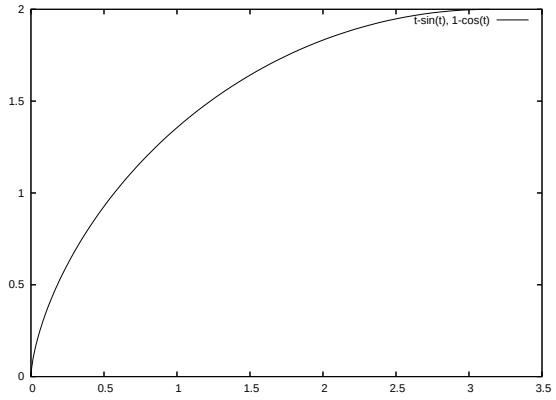

Figure 3: Graph for Problem 4, 2pm.

3 PM

Problem 1

Use the disk method

$$V = \pi \int_2^4 \left(\sqrt{x+1}\right)^2 dx = \pi \int_2^4 (x+1)dx = \pi \frac{(x+1)^2}{2}\Big|_2^4 = \frac{\pi}{2}(25-9) = \frac{16\pi}{2} = 8\pi.$$

Problem 2

Compute the derivative: $f'(x) = 3$.
Use the formula for length

$$L = \int_1^4 \sqrt{1+3^2}dx = \int_1^4 \sqrt{10}dx = \sqrt{10}x\Big|_1^4 = 3\sqrt{10}.$$

Problem 3

Verify $f \geq g$ on the interval $[2,5]$
$f(x) - g(x) = (2x-1) - (x+1) = 2x - 1 - x - 1 = x - 2 \geq 0$ on $[2,5]$.

$$\begin{aligned} M_x &= \frac{1}{2}\int_2^5 \left[(2x-1)^2 - (x+1)^2\right] dx = \frac{1}{2}\int_2^5 (4x^2 - 4x + 1 - x^2 - 2x - 1)dx \\ &= \frac{1}{2}\int_2^5 (3x^2 - 6x)dx = \frac{1}{2}x^3\Big|_2^5 - \frac{3x^2}{2}\Big|_2^5 \\ &= \frac{125 - 8 - 3(25-4)}{2} = \frac{117 - 63}{2} = \frac{54}{2} = 27. \end{aligned}$$

Problem 4

Plot a few points:

t	0	$\pi/2$	π	$3\pi/2$	2π
x	0	1	2	0	
y	0	$\pi/2 - 1$	π	$3\pi/2 - 1$	2π

The direction of the parametrization is counterclockwise.

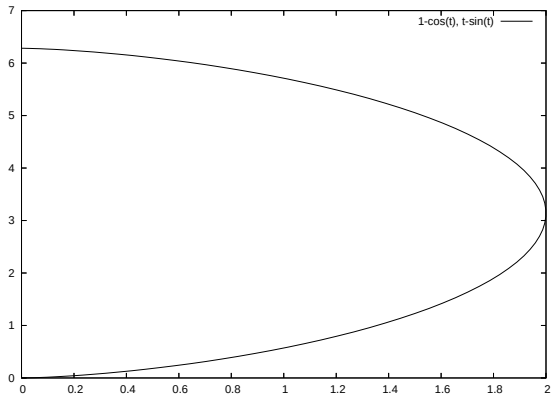

Figure 4: Graph for Problem 4, 3pm.