MATH 410, HW 3

1. Let x_0 be an isolated point of the set D. Prove that every function $f : D \to \mathbb{R}$ is continuous at x_0.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $x = 0$. Prove that

$$\lim_{x \to 0} \frac{f(x^2) - f(0)}{x} = 0.$$

3. Define $f : \mathbb{R} \to \mathbb{R}$ as: $f(x) = x^3$ if $x \in \mathbb{Q}$, and $f(x) = -x^3$ if $x \notin \mathbb{Q}$. Does $f'(0)$ exist? Justify your answer.

4. Let $f : \mathbb{R} \to \mathbb{R}$ be twice differentiable. Assume that for all $x \in \mathbb{R}$, $f(x) \leq 0$ and $f''(x) \geq 0$. Prove that f is constant.

5. Suppose that $f : [a, b] \to \mathbb{R}$ is continuous. For fixed k, let $x_1, \ldots, x_k$ be points in $[a, b]$. Show that there is a point $z \in [a, b]$ at which

$$f(z) = \frac{1}{k}(f(x_1) + \ldots + f(x_k)).$$