MATH 410, HW 4

1. Find the indefinite integral:

$$\int \frac{dx}{\sqrt{(x-a)(x-b)}}.$$

2. Find the indefinite integral:

$$\int \sqrt{\frac{a+x}{a-x}}\, dx.$$

3. Without using the properties of logarithm, prove that for all $x > 0$ and for any rational number $a > 0$:

$$\int_1^{x^a} \frac{dt}{t} = a \int_1^x \frac{dt}{t}.$$

4. Suppose $f : [a, b] \to \mathbb{R}$ is integrable. Prove that

$$\left| \int_a^x f(t)\, dt - \int_a^y f(t)\, dt \right| \leq C|x - y|,$$

for some constant $C > 0$.

5. Suppose that functions $f, g : [a, b] \to \mathbb{R}$ are continuous on $[a, b]$. Prove that

$$\int_a^b |f + g| \leq \int_a^b |f| + \int_a^b |g|.$$