MATH 410, , FALL 2009, Practice MIDTERM I

1. Let A and B be compact subsets of $\mathbb{R}$. Show that their intersection $A \cap B$ is also compact.

2. Let $a \in \mathbb{R}$ be given. Define $S = \{x : x \in \mathbb{Q} \text{ and } x < a\}$. Prove that $a = \sup S$.

3. Using the ϵ-δ "language" prove that $f(x) = \sin(x)$ is continuous.

4. Let $f : [a, b] \to \mathbb{R}$ be continuous and one-to-one and such that $f(a) < f(b)$. Let $c \in (a, b)$. Show that $f(c) \in (f(a), f(b))$.

5. Let $a, b \in \mathbb{R}$, and let $f : (a, b) \to \mathbb{R}$ be uniformly continuous. Show that both $\lim_{x \to a^+} f(x)$ and $\lim_{x \to b^-} f(x)$ exist and are finite numbers.