MATH 411, HW 4

1. Let $f(x,y,z) = x + y + 2z$ be defined on the set $E = \{(x,y,z) \in \mathbb{R}^3 : x > 0, y > 0, z > 0, xyz = 5\}$. Prove that f must assume a minimum value on E and find all points in which this minimum value is assumed.

2. Let $f(x,y,z) = 2x + y^2 + z^3$ be defined on the set $E = \{(x,y,z) \in \mathbb{R}^3 : x > 0, y > 0, z > 0, xyz = 2\}$. Find the point at which f assumes its minium value on E and prove that this indeed is the minimum of f on E.

3. Let a, b, c, k be positive constants. Maximize the expression $x^a y^b z^c$ subject to the restriction $x^k + y^k + z^k = 1$.

4. Let E be the set of points in $\mathbb{R}^3$ which are solutions of the set of 2 equations: $x + y^2 + 3x = 0$ and $x^3 + z^3 + y = 0$. Provide a detailed description of the tangent space to E at the origin.