Solutions for HW3

Problem 1. Evaluate the interpolants at the given points:

$$\lambda_1(x) = \frac{(x-2)(x-3)}{(1-2)(1-3)} = \frac{(x-2)(x-3)}{2}$$

$$\lambda_2(x) = \frac{(x-1)(x-3)}{(2-1)(2-3)} = \frac{(x-1)(x-3)}{-1}$$

$$\lambda_3(x) = \frac{(x-1)(x-2)}{(3-1)(3-2)} = \frac{(x-1)(x-2)}{2}$$

$$y_1 = 2$$

$$y_2 = 5$$

$$y_3 = 4$$

So finally the answer is:

$$f(x) = (x-2)(x-3) - 5(x-1)(x-3) + 2(x-1)(x-2)$$

Problem 2. Note that first three Chebyshev polynomials are:

$$q_0 = 1$$

$$q_1 = x$$

$$q_2 = 2x^2 - 1$$

Writing the approximating sum as follows

$$f(x) = 1 + x^2 = c_0q_0 + c_1q_1 + c_2q_2 = (c_0 - c_2) + c_1x + 2c_2x^2$$

yields the following system:

$$\begin{cases} c_0 - c_2 = 1 \\ c_1 = 0 \\ 2c_2 = 1 \end{cases}$$

which gives us $c_0 = 3/2$, $c_1 = 0$, $c_2 = 1/2$.

Alternatively you may use the explicit formula for finding $c(i)$ using the values of the function at Chebyshev nodes.

Problem 3. Consider the polynomial f of degree n.

First, compute vectors x and f(x):

```
for k=1:n+1
x(k)=cos( pi*(k-1+0.5) / (n+1) );
F(k)=f(x(k));
end;
```

Second, compute the corresponding T_n and $T_n(x)$:

```
for k=1:2
T(k)=cos((k-1)*acos(a));
end;
for k=3:n+1
T(k)=2*a*T(k-1)-T(k-2);
end;
```

Finally, compute the coefficients c_n:

```
c(1)=( 1 / (n+1) )*sum(F);
for k=2:n+1
c(k)=(1 / ( (n+1)/2 ) ) * sum(F.*subs(T(k),a,x));
end;
```

Problem 4. Let $\{x_k\}_{k=1}^{N}$ be the sampling of the interval. The linear approximation on any given subinterval would be:

$$\frac{y_k(x-x_{k-1})+y_{k-1}(x_k-x)}{x_k-x_{k-1}}=\frac{C(x-x_{k-1})+C(x_k-x)}{x_k-x_{k-1}}=C\frac{x_k-x_{k-1}}{x_k-x_{k-1}}=C$$

which exactly equals f.