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Question 1

Trigonometric identity:

$$\begin{aligned} \int \sin^2(x) dx &= \int \frac{1 - \cos(2x)}{2} dx \\ &= \int \frac{1}{2} dx - \int \frac{\cos(2x)}{2} dx \\ &= \frac{x}{2} - \frac{\sin(2x)}{4} + C \end{aligned}$$

Question 2

U substitution, $u = x^3, du = 3x^2 dx$:

$$\begin{aligned} \int x^2 \sin(x^3) dx &= \int \sin(u) \frac{du}{3} \\ &= \frac{1}{3} (-\cos(u)) + C \\ &= \frac{-\cos(x^3)}{3} + C \end{aligned}$$

Question 3

Repeated integration by parts:

$$\begin{aligned} \int \sin(x)e^x &= \sin(x)e^x - \int e^x \cos(x) dx \\ &= \sin(x)e^x - (\cos(x)e^x - \int e^x (-\sin(x)) dx) \\ 2 \int \sin(x)e^x &= \sin(x)e^x - \cos(x)e^x + C \\ \int \sin(x)e^x &= \frac{\sin(x)e^x - \cos(x)e^x}{2} + C \end{aligned}$$

Question 4

U substitution, $u = \cos(x)$, $du = -\sin(x)dx$:

$$\begin{aligned}\int \sin(x) \cos^4(x) dx &= \int -u^4 du \\ &= \frac{-u^5}{5} + C \\ &= \frac{-\cos^5(x)}{5} + C\end{aligned}$$

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Question 1

$$\begin{aligned}\int \cos^2(x) dx &= \int \frac{1 + \cos(2x)}{2} dx \\ &= \int \frac{1}{2} dx + \int \frac{\cos(2x)}{2} dx \\ &= \frac{x}{2} + \frac{\sin(2x)}{4} + C\end{aligned}$$

Question 2

$$\begin{aligned}\int x \cos(x^2) dx &= \int \cos(u) \frac{du}{2} \\ &= \frac{1}{2}(\sin(u)) + C \\ &= \frac{\sin(x^2)}{2} + C\end{aligned}$$

Question 3

Integration by parts:

$$\begin{aligned}\int x \sin(x) dx &= -x \cos(x) - \int -\cos(x) dx \\ &= -x \cos(x) + \sin(x) + C\end{aligned}$$

Question 4

Trigonometric Identity and U substitution, $u = \cos(x)$, $du = -\sin(x)dx$:

$$\begin{aligned} \int \sin^3(x) \cos^2(x) dx &= \int \sin(x)(1 - \cos^2(x)) \cos^2(x) dx \\ &= - \int (1 - u^2)u^2 du \\ &= - \int u^2 - u^4 \\ &= \frac{u^5}{5} - \frac{u^3}{3} + C \end{aligned}$$

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Question 1

$$\begin{aligned} \frac{dy}{dx} &= 1 - y \\ \frac{dy}{y - 1} &= (-1)dx \\ \int \frac{dy}{y - 1} &= \int -1dx \\ \ln |y - 1| &= -x + C \\ y &= De^{(-x)} - 1 \end{aligned}$$

Question 2

$$\begin{aligned} \int x^3 \sin(x^4) dx &= \int \sin(u) \frac{du}{4} \\ &= \frac{1}{4}(-\cos(u)) + C \\ &= \frac{-\cos(x^4)}{4} + C \end{aligned}$$

Question 3

$$\begin{aligned}\int \cos(x)e^{-x} dx &= \cos(x)(-e^{-x}) - \int e^{-x}(-\sin(x))dx \\ &= \cos(x)e^{-x} - (-\sin(x)(-e^{-x}) - \int e^{-x}(-\cos(x))dx) \\ 2 \int \cos(x)e^{-x} &= \cos(x)e^{-x} - \sin(x)e^{-x} + C \\ \int \cos(x)e^{-x} &= \frac{\cos(x)e^{-x} - \sin(x)e^{-x}}{2} + C\end{aligned}$$

Question 4

$$\begin{aligned}\int \sin(x) \cos(x) dx &= \int u du \\ &= \frac{u^2}{2} + C \\ &= \frac{\sin^2(x)}{2} + C\end{aligned}$$

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Question 1

$$\begin{aligned}\frac{dy}{dx} &= 2 + y \\ \frac{dy}{2 + y} &= dx \\ \int \frac{dy}{2 + y} &= \int 1 dx \\ \ln |y + 2| &= x + C \\ y &= De^{-x} - 2\end{aligned}$$

Question 2

$$\begin{aligned} \int x^3 \cos(x^4) dx &= \int \cos(u) \frac{du}{-4} \\ &= \frac{-1}{4} (\sin(u)) + C \\ &= \frac{-\sin(x^4)}{4} + C \end{aligned}$$

Question 3

$$\begin{aligned} \int x^2 \ln(x) dx &= \frac{x^3 \ln(x)}{3} - \int \frac{x^3}{3} \frac{dx}{x} \\ &= \frac{x^3 \ln(x)}{3} - \int \frac{x^2}{3} dx \\ &= \frac{x^3 \ln(x)}{3} - \frac{x^3}{9} + C \end{aligned}$$

Question 4

$$\begin{aligned} \int \sin^2(x) \cos(x) dx &= \int u^2 du \\ &= \frac{u^3}{3} + C \\ &= \frac{\sin^3(x)}{3} + C \end{aligned}$$

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Question 1

$$\begin{aligned} \frac{dy}{dx} &= 1 - 2y \\ \frac{dy}{2y - 1} &= (-1) dx \\ \int \frac{dy}{2y - 1} &= \int -1 dx \\ \frac{\ln |2y - 1|}{2} &= -x + D \\ y &= \frac{De^{-2x} + 1}{2} \end{aligned}$$

Question 2

$$\begin{aligned}\int x^2 \sin(x^3) dx &= \int \sin(u) \frac{du}{3} \\ &= \frac{1}{3}(-\cos(u)) + C \\ &= \frac{-\cos(x^3)}{3} + C\end{aligned}$$

Question 3

$$\begin{aligned}\int x \ln(x) dx &= \frac{x^2 \ln(x)}{2} - \int \frac{x^2}{2} \frac{dx}{x} \\ &= \frac{x^2 \ln(x)}{2} - \int \frac{x}{2} dx \\ &= \frac{x^2 \ln(x)}{2} - \frac{x^2}{4} + C\end{aligned}$$

Question 4

$$\begin{aligned}\int \sin(x) \cos^2(x) dx &= -\int u^2 du \\ &= -\frac{u^3}{3} + C \\ &= \frac{\cos^3(x)}{3} + C\end{aligned}$$

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Question 1

$$\begin{aligned}\int \cos^2(x) dx &= \int \frac{1 + \cos(2x)}{2} dx \\ &= \int \frac{1}{2} dx + \int \frac{\cos(2x)}{2} dx \\ &= \frac{x}{2} + \frac{\sin(2x)}{4} + C\end{aligned}$$

Question 2

$$\begin{aligned}\int 2x^2 \cos(x^3) dx &= 2 \int \cos(u) \frac{du}{3} \\ &= \frac{2}{3} (\sin(u)) + C \\ &= \frac{2 \sin(x^3)}{3} + C\end{aligned}$$

Question 3

$$\begin{aligned}\int \sin(x)e^x &= \sin(x)e^x - \int e^x \cos(x) dx \\ &= \sin(x)e^x - (\cos(x)e^x - \int e^x (-\sin(x)) dx) \\ 2 \int \sin(x)e^x &= \sin(x)e^x - \cos(x)e^x + C \\ \int \sin(x)e^x &= \frac{\sin(x)e^x - \cos(x)e^x}{2} + C\end{aligned}$$

Question 4

$$\begin{aligned}\int \sin^3(x) \cos^2(x) dx &= \int \sin(x)(1 - \cos^2(x)) \cos^2(x) dx \\ &= - \int (1 - u^2)u^2 du \\ &= - \int u^2 - u^4 \\ &= \frac{u^5}{5} - \frac{u^3}{3} + C\end{aligned}$$