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Question 1

$$\begin{aligned} z &= (1+i)^3 \\ &= (\sqrt{2}e^{i\pi/4})^3 \\ &= 2^{3/2}e^{3\pi i/4} \end{aligned}$$

Question 2

$$\begin{aligned} (x^2+y^2)^2 &= x^2 - y^2 \\ ((r\cos\theta)^2 + (r\sin\theta)^2)^2 &= (r\cos\theta)^2 - (r\cos\theta)^2 \\ r^4 &= r^2\cos^2\theta - r^2\sin^2\theta \\ r^2 &= \cos^2\theta - \sin^2\theta \end{aligned}$$

Question 3

One leaf of the graph is given by $0 \leq \theta \leq \pi/4$. Area is

$$\begin{aligned} A &= \int_0^{\pi/4} \frac{1}{2}\sin^2(4\theta)d\theta \\ &= \frac{1}{8}\int_0^{\pi} \sin^2(u)du \\ &= \frac{1}{8}\int_0^{\pi} \frac{1}{2} - \frac{\cos(2u)}{2}du \\ &= \frac{\pi}{16} \end{aligned}$$

Question 4

$$\begin{aligned} x &= 5\cos(\pi/4) = 5\sqrt{2}/2 \\ y &= 5\sin(\pi/4) = 5\sqrt{2}/2 \end{aligned}$$

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Question 1

$$\begin{aligned} z &= (1-i)^3 \\ &= (\sqrt{2}e^{-\pi i/4})^3 \\ &= 2^{3/2}e^{-3\pi i/4} \end{aligned}$$

Question 2

Note $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

$$\begin{aligned}
r &= 2\sin(\theta)\cos(\theta) \\
\sqrt{x^2+y^2} &= 2\sin(\tan^{-1}(y/x))\cos(\tan^{-1}(y/x)) \\
\sqrt{x^2+y^2} &= 2\frac{y}{\sqrt{x^2+y^2}}\frac{x}{\sqrt{x^2+y^2}} \\
(x^2+y^2)^{3/2} &= 2xy
\end{aligned}$$

Question 3

$$\begin{aligned}
x &= 6\cos(3\pi/4) = -6\sqrt{2}/2 = -3\sqrt{2} \\
y &= 6\sin(3\pi/4) = 6\sqrt{2}/2 = 3\sqrt{2}
\end{aligned}$$

Question 4

The graph given is a circle of radius 2. The length of the circle is the diameter. $D = 2\pi r = 4\pi$.

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Question 1

$$z = (2i)^5 = 32i^5 = 32i = 32e^{\pi i/2}$$

Question 2

$$\begin{aligned}
(x^2+y^2)^2 &= x^2 - y^2 \\
((r\cos\theta)^2 + (r\sin\theta)^2)^2 &= (r\cos\theta)^2 - (r\cos\theta)^2 \\
r^4 &= r^2\cos^2\theta - r^2\sin^2\theta \\
r^2 &= \cos^2\theta - \sin^2\theta
\end{aligned}$$

Question 3

$$\begin{aligned}
L &= \int_0^\pi \sqrt{(\sin^2(\theta/2))^2 + (\sin(\theta/2)\cos(\theta/2))^2 d\theta} \\
&= \int_0^\pi \sqrt{\sin^2(\theta/2)((\sin^2(\theta/2) + \cos^2(\theta/2))^2 d\theta} \\
&= \int_0^\pi \sin(\theta/2) d\theta \\
&= -2\cos(\pi/2) + 2\cos(0) \\
&= 2
\end{aligned}$$

Question 4

$$\begin{aligned} x &= 4\cos(-\pi/3) = 4\frac{1}{2} = 2 \\ y &= 4\sin(-\pi/3) = 4\frac{-\sqrt{3}}{2} = -2\sqrt{3} \end{aligned}$$

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Question 1

$$\begin{aligned} z &= (-1-i)^2 \\ &= (\sqrt{2}e^{\frac{5\pi i}{4}})^2 \\ &= 2e^{\frac{10\pi i}{4}} \\ &= 2e^{\pi i/2} \end{aligned}$$

Question 2

Note $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

$$\begin{aligned} r &= 6\sin(\theta)\cos(\theta) \\ \sqrt{x^2+y^2} &= 6\sin(\tan^{-1}(y/x))\cos(\tan^{-1}(y/x)) \\ \sqrt{x^2+y^2} &= 6\frac{y}{\sqrt{x^2+y^2}}\frac{x}{\sqrt{x^2+y^2}} \\ (x^2+y^2)^{3/2} &= 6xy \end{aligned}$$

Question 3

$$\begin{aligned} A &= \int_{-\ln 3}^{0} \frac{1}{2}e^{2\theta}d\theta \\ &= \frac{1}{4}e^0 - e^{-2\ln(3)} \\ &= \frac{1}{4} - 3^{-2} \\ &= \frac{5}{36} \end{aligned}$$

Question 4

$$\begin{aligned} x &= 3\cos(5\pi/6) = 3\frac{-\sqrt{3}}{2} = -\frac{3\sqrt{3}}{2} \\ y &= 3\sin(5\pi/6) = 3\frac{1}{2} = 3/2 \end{aligned}$$

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Question 1

$$\begin{aligned} z &= (3+3i)^2 \\ &= (\sqrt{18}e^{\pi i/4})^2 \\ &= 18e^{2\pi i/4} \\ &= 18e^{\pi i/2} \end{aligned}$$

Question 2

$$\begin{aligned} (r\cos(\theta))^2 + (r\sin(\theta))^2 &= 3r\cos(\theta) + 1 \\ r^2 &= r\cos(\theta) + 1 \end{aligned}$$

Question 3

$$\begin{aligned} x &= 4\cos(-5\pi/6) = 4\frac{-\sqrt{3}}{2} = -2\sqrt{3} \\ y &= 4\sin(-5\pi/6) = 4\frac{-1}{2} = -2 \end{aligned}$$

Question 4

The graph given is a circle of radius 2. The length of the circle is the diameter. $D = 2\pi r = 4\pi$.

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Question 1

$$\begin{aligned} z &= (2-2i)^2 \\ &= (\sqrt{8}e^{\frac{-\pi i}{4}})^2 \\ &= 8e^{\frac{-\pi i}{2}} \end{aligned}$$

Question 2

Note $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

$$\begin{aligned} (x^2+y^2)^2 &= 2x^2 - 2y^2 \\ ((r\cos\theta)^2 + (r\sin\theta)^2)^2 &= 2(r\cos\theta)^2 - 2(r\cos\theta)^2 \\ r^4 &= 2r^2\cos^2\theta - 2r^2\sin^2\theta \\ r^2 &= 2\cos^2\theta - 2\sin^2\theta \end{aligned}$$

Question 3

$$
\begin{aligned}
A &= \int_{-\pi/8}^{\pi/8} \frac{1}{2}(2\cos(4\theta))^2 d\theta \\
&= \int_0^{\pi/8} 4\cos^2(4\theta) d\theta \\
&= \int_0^{\pi/2} \cos^2(u) du \\
&= \int_0^{\pi/2} \frac{1}{2} + \frac{\cos(2u)}{2} du \\
&= \pi/4 + \frac{1}{4}\int_0^{\pi} \cos(v) dv \\
&= \pi/4
\end{aligned}
$$

Question 4

$$
\begin{aligned}
x &= 6\cos(3\pi/4) = 6\frac{-\sqrt{2}}{2} = -3\sqrt{2} \\
y &= 6\sin(3\pi/4) = 6\frac{\sqrt{2}}{2} = 3\sqrt{2}
\end{aligned}
$$