MATH 141, FALL 2015
COMPOSITIONS OF TRIGONOMETRIC FUNCTIONS
WITH INVERSE TRIGONOMETRIC FUNCTIONS

We shall compute a series of compositions of a trigonometric function with an inverse trigonometric function. Our examples that we shall deal with are:

$$\sin(\cos^{-1}(x)), \cos(\cos^{-1}(x)), \tan(\cos^{-1}(x)), \cot(\cos^{-1}(x)), \sec(\cos^{-1}(x)), \csc(\cos^{-1}(x)).$$

As we can see, all of the above examples share a common inverse trigonometric function $\cos^{-1}(x)$. This implies that, if we let $y = \cos^{-1}(x)$, then $x = \cos(y)$. We can thus set a right triangle with one of the acute angles y, its adjacent (denoted by A) equal to x, and hypothenuse (denoted by H) equal to 1. By Pythagorean theorem, we thus have that the side opposite (denoted by O) to angle y must necessarily be $\sqrt{1-x^2}$.

By the definitions of trigonometric functions we thus have:

$$\begin{aligned}
\sin(\cos^{-1}(x)) &= \frac{O}{H} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2},\\
\cos(\cos^{-1}(x)) &= \frac{A}{H} = \frac{x}{1} = x,\\
\tan(\cos^{-1}(x)) &= \frac{O}{A} = \frac{\sqrt{1-x^2}}{x},\\
\cot(\cos^{-1}(x)) &= \frac{A}{O} = \frac{x}{\sqrt{1-x^2}},\\
\sec(\cos^{-1}(x)) &= \frac{H}{A} = \frac{1}{x},\\
\csc(\cos^{-1}(x)) &= \frac{H}{O} = \frac{1}{\sqrt{1-x^2}}.
\end{aligned}$$