

# Math 141 Midterm 1 Question 1 Solution

September 23, 2015

**Question:** Find the volume of the solid obtained by revolving about the  $y$  axis the region between the graphs of functions  $f(x) = \sqrt{3+x}$  and  $g(x) = \frac{\ln(x)}{x^2}$  on the interval  $[1, 3]$ .

**Solution:**

Since we revolve around the  $y$  axis and have our equations in terms of  $x$  we will use the shell method. We see that on  $[1, 3]$

$$f(x) > g(x) \quad 3 \text{ pts}$$

thus

$$V = \int_1^3 2\pi x \left( \sqrt{3+x} - \frac{\ln(x)}{x^2} \right) dx \quad 7 \text{ pts}$$

Pull the  $2\pi$  out and integrate as two separate integrals. For  $\int_1^3 x\sqrt{x+3} dx$  set  $u = 3+x$  so  $du = dx$  and  $x = u-3$ . For  $\int_1^3 \frac{\ln(x)}{x} dx$  set  $w = \ln(x)$  so  $dw = \frac{1}{x} dx$ . Plug the substitutions into the bounds and we have

$$2\pi \left[ \int_4^6 (u-3)\sqrt{u} du - \int_0^{\ln(3)} w dw \right]. \quad 5 \text{ pts}$$

Integrating now gives

$$2\pi \left[ \frac{2}{5} u^{\frac{5}{2}} - 2u^{\frac{3}{2}} \Big|_4^6 - \frac{1}{2} w^2 \Big|_0^{\ln(3)} \right]. \quad 7 \text{ pts}$$

Finally plugging in our bounds gives

$$V = 2\pi \left[ \frac{2}{5} 6^{\frac{5}{2}} - 2 * 6^{\frac{3}{2}} - \frac{2}{5} 4^{\frac{5}{2}} + 2 * 4^{\frac{3}{2}} - \frac{1}{2} [\ln(3)]^2 \right] \quad 3 \text{ pts}$$

and we'll leave our answer as that.