Problem 4 Solution

Question :

Find the particular solution of the following differential equation:

$$\frac{dy}{dx} + sin(x) = ysin(x),$$

with the condition that $y(0) = 1$.

Solution :

First recognize that the differential equation is a first order linear and put it into standard form

$$\frac{dy}{dx} - ysin(x) = sin(x). \qquad \text{(3 pts)}$$

Then find the integrating factor, which is

$$e^{\int -sin(x)\,dx} = e^{cos(x)}. \qquad \text{(5 pts)}$$

Multiple through by the integrating factor and simplify the left hand side.

$$\begin{aligned} e^{cos(x)}\frac{dy}{dx} - ye^{cos(x)}sin(x) &= -e^{cos(x)}sin(x) \\ \frac{d}{dx}\left(ye^{cos(x)}\right) &= -e^{cos(x)}sin(x) \qquad \text{(7 pts)} \end{aligned}$$

integrate both sides

$$ye^{cos(x)} = -\int e^{cos(x)}sin(x)\,dx$$

and use a u-substition on the right hand side

$$\begin{aligned} u &= cos(x) \\ dx &= -sin(x)\,dx, \end{aligned}$$

to get

$$ye^{cos(x)} = e^{cos(x)} + C. \qquad \text{(8 pts)}$$

Then use the initial condition $x = 0$, $y = 1$ to find the value $C = 0$, so the final answer is

$$y = 1. \qquad \text{(2 pts)}$$