Exam 2 Problem 1:

$$\frac{dy}{dx} = \frac{\log_2 x}{x} - \frac{y}{x} \Leftrightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{\log_2 x}{x}$$

is a first linear non-separable differential equation of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where $P(x) = \frac{1}{x}$ and $Q(x) = \frac{\log_2 x}{x}$, so multiplying by the integrating factor

$$e^{\int P(x)dx} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$$

yields

$$(xy)' = x \cdot \frac{\log_2 x}{x} \Rightarrow xy = \int \log_2(x)dx$$

To evaluate $\int \log_2 xdx$, we use integration by parts with the following substitutions $u = \log_2 x$, $dv = dx$, $du = \frac{1}{x\ln 2}dx$, $v = x$, so

$$\int \log_2 xdx = x\log_2 x - \int x\frac{1}{x\ln 2}dx = x\log_2 x - \frac{x}{\ln 2} + C.$$

Thus

$$xy = x\log_2 x - \frac{x}{x\ln 2} + C \Rightarrow y = \log_2 x - \frac{1}{\ln 2} + \frac{C}{x}.$$

- **Either** 8 points for $y = (e^{\int \frac{1}{x}dx}) \int \frac{\log_2 x}{x}(e^{\int \frac{1}{x}dx})dx$
  Or
  - 2 points for writing in normal form
  - 3 points for recognizing $e^{\int \frac{1}{x}dx}$ as integrating factor
  - 3 points for writing $(y \cdot e^{\int \frac{1}{x}dx})' = (e^{\int \frac{1}{x}dx}) \cdot \frac{\log_2 x}{x}$
- 15 points for correct integration
  - 2 points for simplifying to $\log_2 x$
  - 3 points for correctly choosing functions for integration by parts
  - 8 points for using the correct formula (4 points) and computing correctly (4 points)
  - 2 points for constant of integration
- 2 points for simplifying and obtaining the correct solution