MATH 141, FALL 2013, MIDTERM 3
Problem 1

i)

$$\int_0^{\frac{\pi}{2}} \sin^2 t \cos^3 t dt = \int_0^{\frac{\pi}{2}} \sin^2 t \cos^2 t \cos t dt \tag{1}$$

(3 pt.)

$$= \int_0^{\frac{\pi}{2}} \sin^2 t (1 - \sin^2 t) \cos t dt \tag{2}$$

(3 pt.)

$$= \int_0^1 u^2 (1 - u^2) du \tag{3}$$

(2 pt.)

$$= \int_0^1 (u^2 - u^4) du = \frac{u^3}{3} - \frac{u^5}{5}\Big|_0^1 = \frac{1}{3} - \frac{1}{5} = \frac{2}{15} \tag{4}$$

(2 pt.)

i)

$$\int \frac{dx}{\sqrt{4 + 9x^2}} = \int \frac{\frac{2}{3} \sec^2 u du}{2 \sec u} = \frac{1}{3} \int \sec u du \tag{5}$$

(7 pt.)

$$= \frac{1}{3} \ln |\sec u + \tan u| + C \tag{6}$$

(5 pt.)

$$= \frac{1}{3} \ln |\frac{3}{2} \sqrt{x^2 + \frac{4}{9}} + \frac{3}{2} x| + C \tag{7}$$

(3 pt.)