Problem 1 Solution

Question :

Determine if the following improper integral converges or diverges. If it converes, determine its value:

$$\int_0^\infty sin(2x)e^{-x}\,dx.$$

Solution :

Starting with the indefinite integral, do integration by parts with

$$\begin{aligned} u &= sin(2x) & dv &= e^{-x}\,dx \\ du &= 2cos(2x)\,dx & v &= -e^{-x}. \end{aligned} \qquad (5\ \text{pts})$$

$$\int sin(2x)e^{-x}\,dx = -e^{-x}sin(2x) + 2\int e^{-x}cos(2x)\,dx$$

For integral on the right-hand side of the equation do integration by parts with

$$\begin{aligned} u &= cos(2x) & dv &= e^{-x}dx \\ du &= -2sin(2x)\,dx & v &= -e^{-x}. \end{aligned} \qquad (5\ \text{pts})$$

$$\begin{aligned} \int sin(2x)e^{-x}\,dx &= -e^{-x}sin(2x) + 2\left(-cos(2x)e^{-x} - 2\int e^{-x}sin(2x)\,dx\right) \\ &= -e^{-x}sin(2x) - 2cos(2x)e^{-x} - 4\int e^{-x}sin(2x)\,dx \end{aligned}$$

Then solve algebraically for the integral in question.

$$\int sin(2x)e^{-x}\,dx = \frac{1}{5}\left(-e^{-x}sin(2x) - 2e^{-x}cos(2x)\right) \qquad (5\ \text{pts})$$

Then go back to the improper integral and write it as a limit.

$$\begin{aligned} \int_0^\infty sin(2x)e^{-x}\,dx &= \lim_{a\to\infty} \frac{1}{5}\left(-e^{-a}sin(2a) - 2e^{-a}cos(2a)\right) - \frac{1}{5}(0-2) & (4\ \text{pts}) \\ &= 0 - \frac{1}{5}(-2) = \frac{2}{5} & (6\ \text{pts}) \end{aligned}$$