Math 141 Midterm 1 Question 2 Solution

September 24, 2014

Question: Find the largest set which contains 0 and on which the funcion $f(x) = x^9 + 6x^3 + 2x - 10$ has an inverse. Find the derivative of f^{-1} (the inverse of f) at $y = -10$.

Solution:

Take the derivative first

$$f'(x) = 9x^8 + 18x^2 + 2 \qquad \text{4 pts}$$

See that

$$f'(x) > 0 \qquad \text{5 pts}$$

and thus $f(x)$ is always increasing.
Thus the largest set which contains 0 is

$$(-\infty, \infty). \qquad \text{5 pts}$$

To find $(f^{-1})'(-10)$ use the formula

$$(f^{-1})'(c) = \frac{1}{f'(a)}. \qquad \text{5 pts}$$

where f(a)=c.
By inspection find a such that $f(a) = -10$

$$-10 = a^9 + 6a^3 + 2a - 10 \qquad \text{4 pts}$$

so $a = 0$.
Therefore we have

$$(f^{-1})'(-10) = \frac{1}{f'(0)} = \tfrac{1}{2}. \qquad \text{2 pts}$$