Math 141 Midterm 2 Question 2 Solution

October 16, 2015

Question: Evaluate the indefinite integrals:

$$\int x\ \log_{10}(x)\,dx$$

and

$$\int \sin^{-1}(x)\,dx.$$

Solution:
Firstly, if we desire we can rewrite $\log_{10}(x) = \frac{\ln(x)}{\ln(10)}$ and then have $\frac{1}{\ln(10)}\int x\ln(x)\,dx$. Otherwise begin integration by parts:

$$u = \log_{10}(x) \quad dv = x\,dx \qquad du = \frac{1}{x\ln(10)}\,dx \quad v = \frac{x^2}{2} \qquad \text{6 pts}$$

Thus

$$\int x\ \log_{10}(x) = \frac{1}{\ln(10)}\left[\frac{x^2\ln(x)}{2} - \tfrac{1}{2}\int x\,dx\right] \qquad \text{3 pts}$$

and so

$$\int x\ \log_{10}(x) = \frac{1}{\ln(10)}\left[\frac{x^2\ln(x)}{2} - \tfrac{1}{4}x^2 + C\right] \qquad \text{1 pt}$$

Now for $\int \sin^{-1}(x)\,dx$.. Integrate by parts with

$$u = \sin^{-1}(x) \quad dv = dx \qquad du = \frac{1}{\sqrt{1-x^2}}\,dx \quad v = x \qquad \text{5 pts}$$

Therefore

$$\int \sin^{-1}(x)\,dx = x\sin^{-1}(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx. \qquad \text{3 pts}$$

Perform a u-substitution for the resulting integral

$$u = 1 - x^2 \quad du = -2x\,dx \qquad \text{4 pts}$$

and so we have

$$\int \sin^{-1}(x)\,dx = x\sin^{-1}(x) + \tfrac{1}{2}\int \frac{du}{\sqrt{u}} \qquad \text{2 pts}$$

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Our answer is thus

$$\int \sin^{-1}(x)\,dx = x\sin^{-1}(x) + \sqrt{1-x^2} + C \qquad \text{1 pt}$$

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