MATH 141 - Exam 3, Problem 2

Solution

1. (20 points) Find the sum of the following series.

$$\sum_{n=3}^{\infty}\left(\frac{1}{n^3}-\frac{1}{(n+1)^3}\right)$$

Solution: This is a telescoping series. The partial sum is given by

$$s_j = \frac{1}{3^3}-\frac{1}{4^3}+\frac{1}{4^3}-\frac{1}{5^3}+\cdots+\frac{1}{((j+2)^3}-\frac{1}{(j+3)^3}=\frac{1}{3^3}-\frac{1}{(j+3)^3}$$

So, the infinite sum is

$$\sum_{n=3}^{\infty}\left(\frac{1}{n^3}-\frac{1}{(n+1)^3}\right)=\lim_{j\to\infty} s_j=\lim_{j\to\infty}\left(\frac{1}{3^3}-\frac{1}{(j+3)^3}\right)=\frac{1}{27}$$

How points are distributed:

1. Use telescoping - 5pts
2. Correct partial sum - 12pts
3. Get $1/3^3$ - 17pts
4. Get 1/27 (rather than 1/9) - 20pts