

MATH141 - Mid Term Exam 3 - Problem 2

(a)

Substitute: $x - \frac{1}{2} = \frac{1}{2} \sin \theta \implies dx = \frac{1}{2} \cos \theta d\theta$ 2 + 1

$$\begin{aligned}
 \int \sqrt{x-x^2} dx &= \int \sqrt{\frac{1}{4} - \left(x - \frac{1}{2}\right)^2} dx && \text{3} \\
 &= \int \sqrt{\frac{1}{4} - \left(\frac{1}{2} \sin \theta\right)^2} \frac{1}{2} \cos \theta d\theta && \text{1} \\
 &= \frac{1}{4} \int \sqrt{1 - \sin^2 \theta} \cos \theta d\theta \\
 &= \frac{1}{4} \int \cos^2 \theta d\theta && \text{1} \\
 &= \frac{1}{8} \int (1 + \cos 2\theta) d\theta && \text{2} \\
 &= \frac{1}{8} \left(\theta + \frac{\sin 2\theta}{2} \right) + C && \text{1} + \text{1} + \text{1} \\
 &= \frac{1}{8} (\theta + \sin \theta \cos \theta) + C \\
 &= \frac{1}{8} (\sin^{-1}(2x - 1) + 2(2x - 1)\sqrt{x - x^2}) + C && \text{1} + \text{1}
 \end{aligned}$$

(b)

Substitute: $y = e^x \implies dy = e^x dx$ 2 + 2

$$\begin{aligned}
 \int \frac{e^x}{e^{2x} + 3e^x + 2} dx &= \int \frac{1}{y^2 + 3y + 2} dy && \text{1} \\
 &= \int \frac{1}{(y+2)(y+1)} dy && \text{1} \\
 &= \int \frac{1}{y+1} dy + \int \frac{-1}{y+2} dy && \text{2} + \text{2} \\
 &= \ln|y+1| - \ln|y+2| + C && \text{1} + \text{1} + \text{1} \\
 &= \ln(e^x + 1) - \ln(e^x + 2) + C && \text{1} + \text{1} \\
 &= \ln \left(\frac{e^x + 1}{e^x + 2} \right) + C
 \end{aligned}$$