The tank has parabolic cross sections by design, since we obtain the tank by rotating the parabola $y = 4x^2$ around the x-axis for $x \in [0, 1]$. Since $4x^2$ is strictly increasing on this interval, the tank height occurs at the right-hand endpoint, so $h = 4(1)^2 = 4$. The water is being pumped from a full tank to the top of the tank, and we are stopping when there is one foot of water left. The standard equation for work is so far [1]

$$W = 62.5 \int_1^4 (4 - y)A(y)dy$$

We need to compute the area of a given y-cross section. Since $y = 4x^2$, we have $x = \sqrt{y}/2$. The cross sections are clearly circular, and the radius is x, so $A(y) = \pi x^2 = \frac{\pi y}{4}$. This gives the integral

$$W = 62.5 \int_1^4 (4 - y)\left(\frac{\pi y}{4}\right) dy = 62.5 \cdot \frac{9\pi}{4}$$

The integral itself is not hard to evaluate, so most of the points were in setting up the problem. The points were distributed as follows:

1. 5 points for having the correct formula for work.
2. 5 points for finding the tank height.
3. 5 points for having the correct limits of integration.
4. 5 points for computing the cross-sectional area correctly.
5. 5 points for evaluating the integral.

For each of these, I wrote $+k$ near the relevant part of your work if you got k points for that part. But note that I interpret "having the correct formula for work" to mean that you understand the formula. Thus if you know the formula has an $(\ell - x)$ factor, I expect you to know what ℓ is. Also, if you did a volume integral to find the volume of the tank, that suggested you didn't understand the formula, since the formula uses cross-sectional area and not total volume of the tank.

Common mistakes were to switch x and y for only some parts of the problem. For example, many students wrote down the correct formula for work with x and y switched, but had the cross-sectional area as πx^2, which should have really been πy^2 if x and y were switched.

Another common mistake was to compute work for a hemispherical tank rather than a tank with parabolic cross-sections. If you wrote $A(y) = \sqrt{r^2 - y^2}$ for some value of r, this is what you did.

[1] Note that we are integrating with respect to y since the setup of the problem forced y to be the vertical axis. If you like to integrate with respect to x, we can change the x's and y's as long as we remember to rotate the function $x = 4y^2$ around the x-axis.