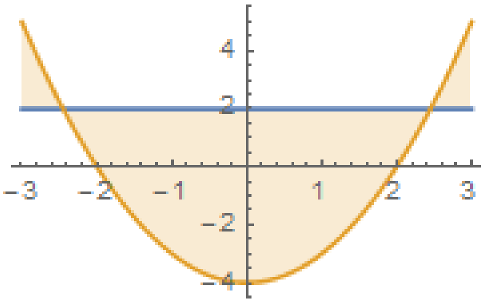
Exam 1 Problem 3:

$$f(x) = g(x) \Leftrightarrow 2 = (x-2)(x+2) = x^2 - 4 \Leftrightarrow x^2 - 6 = 0 \Leftrightarrow x = \pm\sqrt{6}$$

so the region takes x-values in between $-\sqrt{6}$ and $\sqrt{6}$. $2 > g(0) = -4$, so $f > g$ on this region.



Then the area of the region is

$$A = \int_{-\sqrt{6}}^{\sqrt{6}} f(x) - g(x)dx = \int_{-\sqrt{6}}^{\sqrt{6}} 2 - (x^2-4)dx = 2\int_0^{\sqrt{6}} 6 - x^2 dx = 12(\sqrt{6}) - (\frac{1}{3})(12)(\sqrt{6}) = 12(\frac{2}{3})\sqrt{6} = 8\sqrt{6}$$

$$M_y = \int_{-\sqrt{6}}^{\sqrt{6}} x(6-x^2)dx = \int_{-\sqrt{6}}^{\sqrt{6}} 6x - 6x^3 dx = 0$$

since $6x - 6x^3$ is an odd function.
(This is also given by the fact that $f(x) = f(-x)$ and $g(x) = g(-x)$ means that f and g have y-axis symmetry.)

$$M_x = \frac{1}{2}\int_{-\sqrt{6}}^{\sqrt{6}} 2^2 - (x^2-4)^2 dx = 2(\frac{1}{2})\int_0^{\sqrt{6}} 4 - (x^4 - 8x^2 + 16)dx =$$

$$= \int_0^{\sqrt{6}} -x^4 + 8x^2 - 12 dx = -\frac{1}{5}(36)\sqrt{6} + \frac{8}{3}(6)\sqrt{6} - 12\sqrt{6} = -\frac{16}{5}\sqrt{6}$$

Thus

$$\bar{x} = \frac{M_y}{A} = \frac{0}{A} = 0$$

and

$$\bar{y} = \frac{M_x}{A} = (-\frac{16\sqrt{6}}{5})(\frac{1}{8\sqrt{6}}) = -\frac{2}{5}.$$

5 points for computing area: 2 points for correct integral setup, 3 points for correct computation, -1 point per mistake

8 points for finding M_x: 2 points for correct integral setup, 6 points for correct computation, -1 point per mistake

8 points for EITHER computing M_y OR using symmetry to determine $M_y = 0$:
2 points for correct integral setup, 6 points for correct computation, -1 point per mistake OR 2 points for writing that that there is symmetry about $x = 0$, 6 points for explanation (detailed labeled graph or showing f and g are even functions)

4 points for final answer: 1 point each for noting $\bar{x} = \frac{M_y}{A}$, $\bar{y} = \frac{M_x}{A}$, 1 point for each simplified correct value