Math 141 Midterm 1 Question 2 Solution

October 13, 2014

Question: Evaluate the following definite integrals:

a)

$$\int_0^1 4^{3x}\, dx;$$

b)

$$\int_1^2 \frac{1}{x(4 + \ln^2(x))}\, dx.$$

Solution:

a) Set $u = 3x$, $du = dx$ so we have $\frac{1}{3}\int 4^u du$. Integrate to get

$$\frac{1}{3\ln(4)}\, 4^{3x} \qquad\qquad \text{7 pts}$$

Plug in bounds:

$$\frac{1}{3\ln(4)}(4^3 - 4^0) = \frac{63}{3\ln(4)} = \frac{21}{\ln(4)} \qquad\qquad \text{3 pts}$$

b) Set $u = \ln(x)$, $du = \frac{1}{x}dx$ so we have

$$\int \frac{1}{4 + u^2}\, dx \qquad\qquad \text{5 pts}$$

Recognize this is the derivative of an inverse tangent function so

$$\int \frac{1}{4 + u^2}\, dx = \tfrac{1}{2}\tan^{-1}(\tfrac{u}{2}) = \tfrac{1}{2}\tan^{-1}(\tfrac{\ln(x)}{2}) \qquad\qquad \text{5 pts}$$

Plug in bounds:

$$\tfrac{1}{2}(\tan^{-1}(\tfrac{\ln(e^2)}{2}) - \tan^{-1}(\tfrac{\ln(1)}{2})) = \tfrac{1}{2}(\tan^{-1}(1) - \tan^{-1}(0)) = \tfrac{1}{2}(\tfrac{\pi}{4} - 0) = \tfrac{\pi}{8} \qquad\qquad \text{5 pts}$$