Problem 4 Solution

Question :

Let R be the region between the graph of function $f(x) = 2 + \sqrt{1 - x^2}$ and the x-axis, on the interval $[0, 1]$. Find the volume of the solid obtained by revolving R about the y-axis.

Solution :

First set up the formula for the volume using the shells method

$$V = 2\pi \int_0^1 x \left(2 + \sqrt{1 - x^2}\right) dx. \qquad \text{(10 pts)}$$

Then integrate the first half of the definite integral

$$2\pi \int_0^1 2x \, dx = 2\pi x^2 \Big|_0^1 = 2\pi. \qquad \text{(3 pts)}$$

To integrate $2\pi \int_0^1 x\sqrt{1 - x^2}\, dx$ use a u substitution

$$\begin{aligned} u &= 1 - x^2 \\ du &= -2x \, dx \end{aligned} \qquad \text{(5 pts)}$$

which gives

$$2\pi \int_{x=0}^{x=1} x\sqrt{1 - x^2}\, dx = -\pi \int_{u=1}^{u=0} \sqrt{u}\, du. \qquad \text{(5 pts)}$$

So the final answer is the sum of the two halves of the definite integral

$$V = 2\pi + \left(-\pi \frac{2}{3} u^{3/2} \Big|_1^0\right) = \frac{8\pi}{3}. \qquad \text{(2 pts)}$$