

Solution: Exam 2, Problem 4

18th October 2013

Grading scheme 1:

$$L = \lim_{x \rightarrow \infty} \frac{\log_2(\log_3 x)}{\log_5 x}$$

$$L = \left(\frac{\ln 5}{\ln 2}\right) \lim_{x \rightarrow \infty} \frac{\ln(\log_3 x)}{\ln x} \dots (5 \text{ points})$$

As $x \rightarrow \infty$, $L \rightarrow \frac{\infty}{\infty}$ which is indeterminate, so use the L'Hospital's Rule ... (5 points)

$$L = \left(\frac{\ln 5}{\ln 2}\right) \lim_{x \rightarrow \infty} \frac{\frac{1}{\log_3 x} \frac{\partial \log_3 x}{\partial x}}{\frac{1}{x}} \dots (5 \text{ points})$$

$$L = \left(\frac{\ln 5}{\ln 2}\right) \lim_{x \rightarrow \infty} \left(\frac{\ln 3}{\ln x}\right)(x) \left(\frac{1}{x}\right) \left(\frac{1}{\ln 3}\right) = \left(\frac{\ln 5}{\ln 2}\right) \lim_{x \rightarrow \infty} \left(\frac{1}{\ln x}\right) = \left(\frac{\ln 5}{\ln 2}\right) \left(\frac{1}{\infty}\right) \dots (5 \text{ points})$$

Thus, $L = 0 \dots (5 \text{ points})$

Grading scheme 2:

$$L = \lim_{x \rightarrow \infty} \frac{\log_2(\log_3 x)}{\log_5 x}$$

As $x \rightarrow \infty$, $L \rightarrow \frac{\infty}{\infty}$ which is indeterminate, so use the L'Hospital's Rule ... (5 points)

$$L = \lim_{x \rightarrow \infty} \frac{\left(\frac{1}{\log_3 x}\right) \left(\frac{1}{\ln 2}\right) \left(\frac{1}{x \ln 3}\right)}{\frac{1}{x \ln 5}} \dots (10 \text{ points})$$

$$L = \lim_{x \rightarrow \infty} \frac{x \ln 5}{(\log_3 x)(\ln 2)(x \ln 3)} = \lim_{x \rightarrow \infty} \frac{\ln 5}{(\log_3 x)(\ln 2)(\ln 3)} = \lim_{x \rightarrow \infty} \frac{(\ln 5)(\ln 3)}{(\ln x)(\ln 2)(\ln 3)} \dots (5 \text{ points})$$

Thus, $L = 0 \dots (5 \text{ points})$