Exam 3

Grading key to problem 4

4)

a) The general formula for the fifth degree Taylor polynomial for f(x) is

$P_5(x) = f(0) + f`(0)x + \frac{1}{2!}f``(0)x^2 + \frac{1}{3!}f^{(3)}(0)x^3 + \frac{1}{4!}f^{(4)}(0)x^4 + \frac{1}{5!}f^{(5)}(0)x^5$

Finding the derivatives of $f(x) = e^{-x}$ was worth five points.

Properly evaluating the derivatives at 0 and using the correct formula was worth the other five points.

$P_5(x) = 1 - x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \frac{1}{4!}x^4 - \frac{1}{5!}x^5$

b) Changing to a continuous variable (usually "x") was worth 5 points. Then properly finding the limit was worth 10 points (with justifications for passing the limit through to the argument of the exponential function worth 5 and evaluating correctly worth 5).

Let $f(x) = e^{1/x}$. Note that f(x) is continuous and $f(n) = e^{1/n}$. So, by Theorem 9.4, $lim_{n\to\infty}e^{1/n} = lim_{x\to\infty}e^{1/x}$.

Since exponentiation is a continuous function, we have that

$lim_{x\to\infty}e^{1/x} = e^{lim_{x\to\infty}1/x} = e^0 = 1$.