Solution: Exam 3, Problem 4

10th November 2013

Problem 4 a

Grading scheme 1:

$$L = \lim_{n\to+\infty} \sqrt[n]{n^2} = \lim_{n\to+\infty} n^{\frac{2}{n}}$$

Say $f(x) = x^{\frac{2}{x}}$, then,

$$L = \lim_{n\to+\infty} n^{\frac{2}{n}} = \lim_{x\to+\infty} x^{\frac{2}{x}} = \lim_{x\to+\infty} e^{\frac{2\ln x}{x}}$$

Now, $\lim_{x\to+\infty} \frac{\ln x}{x}$ is of the form $\frac{\infty}{\infty}$, which is indeterminate. Therefore, use the L'Hospital's Rule ... 5 points (till here).

$$\lim_{x\to+\infty} \frac{\ln x}{x} = \lim_{x\to+\infty} \frac{1}{x} = 0$$

Therefore,

$$\lim_{n\to+\infty} n^{\frac{2}{n}} = \lim_{x\to+\infty} e^{2(0)} = 1 \ldots 5 \text{ points (till here)}$$

Problem 4 a

Grading scheme 2:

$$L = \lim_{n\to+\infty} \sqrt[n]{n^2} = \lim_{n\to+\infty} (\sqrt[n]{n})^2 \ldots 3 \text{ points}$$

$$\text{And } \lim_{n\to+\infty} \sqrt[n]{n} = 1 \ldots 5 \text{ points}$$

$$\text{So } \lim_{n\to+\infty} (\sqrt[n]{n})^2 = 1 \ldots 2 \text{ points}$$

Problem 4 b

Grading scheme:

$$L = \lim_{n\to+\infty} (\sqrt{n+1} - \sqrt{n}) = \lim_{n\to+\infty} \frac{(\sqrt{n+1} - \sqrt{n})(\sqrt{n+1} + \sqrt{n})}{(\sqrt{n+1} + \sqrt{n})} \ldots 5 \text{ points}$$

$$L = \lim_{n\to+\infty} \frac{(n+1-n)}{(\sqrt{n+1} + \sqrt{n})} = \lim_{n\to+\infty} \frac{1}{(\sqrt{n+1} + \sqrt{n})} \ldots 3 \text{ points}$$

$$L = \lim_{n\to+\infty} \frac{1}{\sqrt{n}(\sqrt{1+\frac{1}{n}} + 1)} = \lim_{n\to+\infty} \frac{1}{\sqrt{n}} \quad \lim_{n\to+\infty} \frac{1}{(\sqrt{1+\frac{1}{n}} + 1)} = 0(\frac{1}{1+1}) = 0 \ldots 7 \text{ points}$$