MATH141 - Mid Term Exam 4 - Problem 4

$z^3 + 8 = 0 \implies z^3 = -8 = 8e^{i\pi}$ $\boxed{2}$

Put $z = re^{i\theta}$

$z^3 = r^3 e^{i3\theta} = 8e^{i\pi}$ $\boxed{2}$

Thus,

$r^3 = 8$ and $3\theta = \pi + 2k\pi,\ k$ is an integer $\boxed{2}$ $+$ $\boxed{2}$

Which gives, $r = 2$ $\boxed{2}$ and $\theta = \frac{\pi}{3} + \frac{2k\pi}{3}$ $\boxed{3}$ $\implies z = 2e^{i(\frac{\pi}{3} + \frac{2k\pi}{3})}$, k is an integer. $\boxed{3}$

Therefore, there are three distinct solutions corresponding to $k = 0, 1, 2,$

$$z = 2e^{i\frac{\pi}{3}},\ 2e^{i\pi},\ 2e^{i\frac{5\pi}{3}} \quad \boxed{3}$$

$$z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right),\ 2\left(\cos\pi + i\sin\pi\right),\ 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) \quad \boxed{3}$$

$$z = 1 + i\sqrt{3},\ -2 + i0,\ 1 - i\sqrt{3} \quad \boxed{3}$$

$\boxed{-1}$ If all all three final answers are correct but have not shown the intermediate steps.