MATH 141, FALL 2015, Some Trigonometric Integration Examples

When solving trigonometric integrals, we are given 3 different classes of problems to analyze:

(1) Identify the integrals in the form of a function of sin and cos;
(2) Identify the integrals in the form of a function of sec and tan;
(3) Identify the integrals in the form of a function of csc and cot.

For these classes we have specific instructions given. And for integrals which do **not** fit into any of the above scenarios, we are told to comnvert them into a function of sin and cos.

This strategy may sometimes lead us into solving problems, which can be solved much more simply. Please study the examples below.

Example 1. Consider the following integral

$$\int \frac{\tan(x)}{\sec(x)}\,dx.$$

Our algorithm from the textbook indicates that we need to factor out $\sec(x)\tan(x)$, and do a substitution $u = \sec(x)$. So

$$\int \frac{\tan(x)}{\sec(x)}\,dx = \int \frac{\sec(x)\tan(x)}{\sec^2(x)}\,dx = \int \frac{1}{u^2}\,du = -\frac{1}{u} + C = -\frac{1}{\sec(x)} + C.$$

But note that, after simplifying the above integral into a function of sin and cos, we get:

$$\int \frac{\tan(x)}{\sec(x)}\,dx = \int \frac{\sin(x)\cos(x)}{\cos(x)}\,dx = \int \sin(x)\,dx = -\cos(x) + C.$$

Clearly,

$$-\frac{1}{\sec(x)} = -\cos(x),$$

so both solutions are correct, but the latter one is more direct and does not require any integration techniques.

Example 2. An even better example is the following integral

$$\int \frac{\sec(x)}{\tan^2(x)}\,dx.$$

This example belongs to the case where we are instructed to rewite the function as a function of secant and proceed. But this leads us to

$$\int \frac{\sec(x)}{\tan^2(x)}\,dx = \int \frac{\sec(x)}{\sec^2(x)-1}\,dx.$$

It is not obvious what the next step sould be, as the textbook doesn't provide us with any more guidance beyond rewriting the integrand as a function of secant.

However, if we decided to go against the suggestions and to rewrite the integrand as a function of sin and cos, we get the following:

$$\int \frac{\sec(x)}{\tan^2(x)}\,dx = \int \frac{\cos^2(x)}{\sin^2(x)\cos(x)}\,dx = \int \frac{\cos(x)}{\sin^2(x)}\,dx = -\frac{1}{\sin(x)} + C = -\csc(x) + C.$$

Please have these 2 examples in mind when looking at trigonometric integrals.